Dynamic Pricing in a Coffee Shop

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Abstract. We develop an optimization model for dynamic pricing in a coffe shop that seeks to maximize total contribution. Based on the random utility maximization theory, the preference-based demand function is derived from choice data using sequential processes of estimating individual utilities and simulating and aggregating individual choices. Individual utilities are estimated using the hierarchical Bayes method, while individual choices are estimated using the randomized first choice simulation. We implemented the approach in a coffee shop in Jakarta, Indonesia, and considered only one product i.e. milk coffee, which constituted two-third of total sales. To avoid complexity in the implementation, we considered two fare-classes and developed three time-based pricing scenarios. The solution to the mixed-integer nonlinear programming problem was obtained using enumeration. We came up with optimal prices for a cup of milk coffee of Rp15,000 in the morning, and Rp23,000 in other time of day. Using this pricing policy, the monthly total contribution was estimated to increase by 11%, from Rp71.9 millions to Rp79.8 millions.
Keywords: dynamic pricing, preference-based demand function, hierarchical Bayes, randomized first choice.

1. Introduction

Since 2016, coffee consumption in Indonesia has grown rapidly and it was estimated that until 2021 the business will still grow at an average annual rate of 8.22%. At the same time, coffee shops have grown swiftly in many cities in Indonesia, including Jakarta. It was estimated that until mid of 2018 there were about 1,500 coffee shops in Jakarta growing at an annual rate of 10%. This has brought an intense competition to the industry. Pricing policy and tactic through revenue management (RM) initiative are viable options for responding to such situation.

Research on RM in food and beverage (F&B) industry are dominated by those in restaurant and usually focus on duration management [1,2] and demand-based pricing [3,4,5]. Fewer research work on the supply side like table mix [6]. Compared to the traditional RM industries such as hotels and airlines, restaurants have less fixed service capacity, variable duration of service, and more elastic physical constraint [7]. Coffee shops are similar to restaurants except that they have even less fixed service capacity and more variable duration of service. The capacity is less fixed because customers may buy coffee for take-away, while the duration is relatively more variable because some customers tend to stay longer in coffee shops either for individual (e.g. students working on their assignment) or social (e.g. group meeting) activities. In a coffee shop, revenue is generated through selling main products (mostly coffee beverages) and other ancillary products (e.g. cakes, cookies, dishes).

This research aims to develop RM tactics for a coffee shop in Jakarta, Indonesia, called Nusa Kopi. This coffee shop has been facing an intense competition from nearby coffee shops. There are at least four coffee shops at walking distance around this coffee shop. The RM tactics focused on the most basic one, i.e. price differentiation. We seek to determine optimal prices for a single product, i.e. milk coffee, which constitutes two-third of total sales. At the beginning, we conducted interview with the owner to explore the nature of the RM initiatives that he wished to implement. Considering that competitors were adopting a one-fare-class policy for their products, the owner of the coffee shop preferred a simple two-fare-class policy with a possibility of future enhancement depending on the competitors' responses.

The most important input, yet the most difficult to obtain, in pricing optimization is demand function [8]. In our research, the demand function is derived based on the robust theory of random utility maximization [9]. According to this theory, an individual attaches a value (called utility) to each level of each product attribute. In a particular choice situation, this individual will choose product (or option) that has the greatest total utility. In our research, individual utilities are estimated from choice data using hierarchical Bayes (HB) method [10]. Using information about individual utilities, we can predict how each individual will respond to a particular choice situation. Aggregating these individual choices results in the estimated share of preference (SoP), i.e. proportion of population that would choose a particular option in a particular choice situation. To estimate SoP, we used the randomized first choice (RFC) simulation method which capable of alleviating the independence-of-irrelevant-alternatives (IIA) problem [11]. By running the RFC simulation with different price levels, we obtained the so-called SoP data points, i.e. pairs of price-SoP. Multiplying the SoP with the maximum achievable demand resulted in demand data points. In pricing optimization, we need a continuous and explicit demand function. In our research, this was obtained by interpolating the SoP data points using cubic splines.

Most research in pricing optimization assume independent demand where the quantity demanded is not affected by producer's decision on price and product attributes [12]. In multiple-fare-class problem, this implies that price in a particular fare-class does not affect demand in other fare-class. This may be valid for commodity, but not for differentiated product in competitive situation. Our model takes the impact of prices between fare-classes into account. When we analyze a particular fare-class, we consider this fare-class as our product-of-interest and other fare-classes as competitors' products, and vice versa. Demand for our product-of-interest depends on the prices and attributes of competitors' products. This dependence is represented in the scenario of the randomized first choice simulation. This makes our model is more realistic than those assuming independent demand.

2. Methods

Basic uncapacitated pricing optimization problem with single fare-class is one where we are to determine one price that will maximize total contribution. Mathematically, this can be expressed as follows [8]

$$\max_{p} d(p)(p-c) \tag{1}$$

where p is price, $d(p)$ is demand function, and c is incremental cost.

In this research we consider two fare-classes, fare-class 1 which has a higher price and fare-class 2 with a lower price. Since we seek to maximize total contribution, the objective of our optimization problem becomes

$$\max_{p_1,p_2} d_1(p_1)(p_1-c) + d_2(p_2)(p_2-c) \tag{2}$$

The demand function for fare-class 1 and fare-class 2 are represented by $d_1(p_1)$ and $d_2(p_2)$, respectively, where p_1 and p_2 are prices for each fare-class.The incremental costs for both fare-classes are the same (c) because the price differentiation will be based on the time of purchase.

Although Equation 1 and 2 seem to be similar, how we model the demand function will determine the complexity of the second problem. If demand in both fare-classes are assumed to be independent, $d_1(p_1)$ and $d_2(p_2)$ come from single demand function that is segregated based on consumers' willingness-to-pay. Reader may refer to Pratikto [13] for the formulation of this problem. In this research, interdependence of demand between fare-classes are allowed. We derived the demand functions from choice data collected through a discrete choice experiment (DCE) questionnaire. Choice data are responses to a systematically designed set of choice tasks, each of which consists of a number of concepts or products (usually 3 to 4) which represent a particular choice situation. A "none" option may be added in each choice task to mimic the situation where consumers may decide to not to buy any of the available alternatives. Each product is composed of attributes representing its characteristics, e.g. for car the attributes may be brand, model, engine cc, fuel consumption (mpg), and price. Each attribute has several possible values called levels, e.g. mpg may have levels of 5, 7, 9, or 12.

The best method for estimating utilities from choice data is hierarchical Bayes [10] which produces utilities at individual level. We use this method by assuming that individual utilities follow normal multivariate probability distribution. Let $\boldsymbol{\beta}$ be the vector of utilities that are normally distributed such that $\boldsymbol{\beta} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}$ is the mean vector and $\boldsymbol{\Sigma}$ is the covariance matrix. Based on the Bernstein-von Mises theorem, each of those three parameters can be viewed as stochastic processes that evolve with data, and hence estimated iteratively conditional on the other two. The value of $\boldsymbol{\mu}$ conditional on $\boldsymbol{\Sigma}$ and $\boldsymbol{\beta}$ is normally distributed, while the value of $\boldsymbol{\Sigma}$ conditional on $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$ follows inverted Wishart distribution. Meanwhile, the value of $\boldsymbol{\beta}$ is estimated conditional on $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ from the previous iteration.

Once we have $\boldsymbol{\beta}$, we can predict how a consumer responds to a particular choice situation. Let vector $\boldsymbol{\beta}_m$ be the m-th row of matrix $\boldsymbol{\beta}$ which represents the utilities of each level attribute for consumer m. Now let $\mathbf{x}_i$ be a binary vector representing the attribute levels of product i, where 1 represents that a particular level is present and 0 is not. Then, under randomized first choice, total utilities of product i for consumer m, U_{mi} can be calculated as follows [11]

$$U_{mi} = \boldsymbol{x}_i{}'[\boldsymbol{\beta}_m + \boldsymbol{\varepsilon}_m] + \varepsilon_i \tag{3}$$

Two random components are added in U_{mi}, $\boldsymbol{\varepsilon}_m$ which represents error in evaluating attribute levels, and ε_i which represents error in evaluating alternatives. Consumer m will choose product i if and only if

$$U_{mi} \geq U_{mj}, \forall j \neq i \tag{4}$$

Suppose we have choice data of M consumers. Then, by aggregating across all consumers, we can estimate s_i, the share-of-preference of product i, i.e. proportion of consumers that will choose product i in the given choice situation, as follows

$$s_i = \frac{\sum_{m=1}^{M} 1_{U_{mi} \geq U_{nj},\ \forall j \neq i}}{M} \tag{5}$$

Since s_i is a random variable, to obtain its estimate (i.e. $\hat{s}_i$), we need to draw large enough samples using Equation 3 and Equation 5, discard the transient data and calculate the sample mean. Under restrictive condition, $\hat{s}_i$ can be used as a proxy for market share. From Equation 3 we know that U_{mi} depends on $\mathbf{x}_i$ which comprises information about price, denoted by p_i, and non-price attributes, denoted by vector $\mathbf{q}_i$. Then, Equation 5 implies that market share of product i also depends on prices and non-price attributes of competitors' products which can be respectively denoted by $\mathbf{p}_j$ and $\mathbf{q}_j$ for all $j \neq i$. Since we are only interested in the demand as a function of price (not of other attributes), the complete notation for estimated market share of product i as a function of its price is $\hat{s}_i\left(p_i | q_i, \mathbf{p}_j, \mathbf{q}_j\ \forall j \neq i\right)$. If the market size is D then the demand for product i can be estimated as follows

$$d_i(p_i) = D\hat{s}_i\left(p_i | q_i, \boldsymbol{p}_j, \boldsymbol{q}_j\ \forall j \neq i\right) \tag{6}$$

In a two-fare-class problem like ours, when we analyze fare-class 1, fare-class 2 together with other products available in the market are considered as competitors. Hence, in addition to the objective function as in Equation 2, we have the following constraints

$$d_1(p_1) = D\hat{s}_1\left(p_1 | q_1, p_2, q_2, \boldsymbol{p}_j, \boldsymbol{q}_j\ \forall j \neq 1{,}2\right) \tag{7}$$

$$d_2(p_2) = D\hat{s}_2\left(p_2 | q_2, p_1, q_1, \boldsymbol{p}_j, \boldsymbol{q}_j\ \forall j \neq 1{,}2\right) \tag{8}$$

and also the nonnegativity constraint $0 \le p_2 < p_1$. Both $\hat{s}_1(p_1|q_1, p_2, q_2, \mathbf{p}_\text{j}, \boldsymbol{q}_\text{j}\ \forall j \neq 1{,}2)$ and $\hat{s}_2(p_2|q_2, p_1, q_1, \mathbf{p}_j, \mathbf{q}_j\ \forall j \neq 1{,}2)$ and hence $d_1(p_1)$ and $d_2(p_2)$ are not continuous because the possible values of p_1, and p_2 (and also p_j) are only those defined as price levels in the DCE design. To make them continuous, we interpolate between data points using the cubic splines. Reader may refer to Wolberg [14] for a comprehensive discussion on the cubic spline intepolation. This approach results in a smooth demand function and fits with all data points. We consider this better than curve-fitting the data points to a particular theoretical demand function. The latter—although has a relatively simpler explicit function—usually result in a partial-fit demand function.

From Equation 2, 7, and 8 it can be observed that we have an optimization problem in which the parameters of the objective function depend on the values of decision variables. There is no algorithm to find solution to this kind of problem. In this research we use enumeration.

3. Result and Discussion

Since we are to implement time-based price differentiation, in our discrete choice experiment questionnaire we set attributes and levels as in Table 1. The current price was Rp25,000 but we want to explore the price range from as low as Rp10,000 to as high as Rp35,000. The lowest price of Rp10,000 is still feasible because the incremental cost for a cup of milk coffee is only Rp6,600.

Table 1. Attributes and levels

Attributes	Levels
Coffee shop	Nusa Kopi
	Folosofi Kopi
	Kopi Tuku
	Kopi Kalyan
	Anomali Coffee
Time of day	Morning (7 - 11am)
	Rest of the day
Price	Rp10,000
	Rp15,000
	Rp22,000
	Rp28,000
	Rp35,000

Each DCE questionnaire consists of 8 random and 1 fixed choice tasks. In each questionnaire, the random choice tasks were designed such that they are balanced, orthogonal, and have minimum overlap. Since the choice tasks were randomly generated, there are no two respondents that get exactly the same choice tasks. Meanwhile, the fixed choice task was for internal validity test. We used the Sawtooth Software SSI Web v7.0.30 to design and generate the questionnaire and administer the online survey. Our academic license only permits a maximum of 250 data used in the estimation process. Sawtooth Software CBC/HB v5.5.6 were used to estimate the utility values for each respondent.

We tested the internal validity of the utility estimates by comparing the real choice of the fixed task with one predicted based on the utility values. We came up with a mean absolute error of 2.71% which is equivalent to mean absolute percentage error of 10.82%. This means that we will make about 11% error when predicting respondents' choice based on these utility values. This is a good result despite the relatively small sample size.

In the next stage, using information about the utilities we derived the demand function by simulating and aggregating individual choice using the randomized first choice method. We used Sawtooth Software SMRT v4.23.0 to simulate the choices set two scenarios: (1) one fare-class for all time of day; and (2) two fare-class i.e. lower price in the morning and normal price for the rest of the day. Setting in Scenario 2 is based on the observation that there is only a few customers come to the coffee shop in the morning. In Scenario 1, the milk coffee from Nusa Kopi competes with similar products from competitors as depicted in Table 2.

Table 2. Alternatives in Senario 1

Milk coffee from	Prices
Nusa Kopi	Run for all possible price levels
Filosofi Kopi	Rp22,000
Kopi Tuku	Rp15,000
Kopi Kalyan	Rp15,000
Anomali Coffee	Rp22,000

From Scenario 1 we came up with five demand data points as in Table 3.

Table 3. Share-of-preference from Scenario 1

Prices	Share-of-preferences
Rp10,000	21.38%
Rp15,000	20.67%
Rp22,000	22.69%
Rp28,000	12.86%
Rp35,000	2.65%

Normally, we would expect a downward-sloping share-of-preference curve, but we can see in Table 2 that for prices of Rp10,000 and Rp15,000 the share-of-preferences are slightly lower than that of Rp22,000. This fact implies that consumers are not price-sensitive at lower price. It could mean that for some consumers, Rp10,000 and Rp15,000 are considered unreasonable for a cup of milk coffee that they are questioning the quality of the product sold at those price levels. Interpolating between the share-of-preference data points in Table 2, we came up with the following share-of-preference function

$$\hat{s}(p) = \begin{cases} 0.16 + 2.33 \cdot 10^{-5}p - 2.70 \cdot 10^{-9}p^2 + 9.00 \cdot 10^{-14}p^3, & 10000 \le p < 15000 \\ 1.13 + 1.70 \cdot 10^{-4}p + 1.02 \cdot 10^{-8}p^2 - 1.97 \cdot 10^{-13}p^3, & 15000 \le p < 22000 \\ -3.12 + 4.09 \cdot 10^{-4}p - 1.61 \cdot 10^{-8}p^2 + 2.02 \cdot 10^{-13}p^3, & 22000 \le p < 28000 \\ 2.20 - 1.61 \cdot 10^{-4}p + 4.25 \cdot 10^{-9}p^2 - 4.05 \cdot 10^{-14}p^3, & 28000 \le p < 35000 \\ 0, & otherwise \end{cases} \quad (9)$$

For brevity, we use $\hat{s}(p)$ instead of $\hat{s}_i(p_i | q_i, \mathbf{p}_j, \boldsymbol{q}_j \; \forall j \neq i)$. We do not extrapolate beyond the predetermined price ranges since there is no information about respondents' preferences in that region. Using calibration with historical data, we came up with maximum achievable demand, $D = 21{,}160$, while the incremental cost is estimated to be Rp6,600. Plugging the demand function from Equation 9 into pricing optimization in Equation 1 and enumerating the feasible prices in multiple of Rp500, we came up with optimal price of Rp23,000, with estimated total contribution of Rp74,980,800.

In Scenario 2 we seek to determine optimal prices with a scenario as in Table 4. Nusa Kopi 2 refers to milk coffee offered in the morning woth lower price, while Nusa Kopi 1 is one with higher price offered on the rest of the day. In Scenario 2, the time-of-day of competitors are set to "Rest of the day" because they are implementing one-fare-class policy and "Morning" is less desirable than "Rest of the day". In this problem we need to determine optimal prices for Nusa Kopi 2 and Nusa Kopi 1. To make the process of finding the solution easier, we restrict the possible price for Nusa Kopi 2 to those price levels set in the DCE design,i.e. Rp10,000 and Rp15,000. Meanwhile, the possible prices for Nusa Kopi 1 are the same as those of the one-fare-class problem, i.e. from Rp10,000 or Rp15,000 (depending on the price of Nusa Kopi 2) to Rp35,000 in multiple of Rp500.

Equation 10 and Equation 11 shows the share-of-preference function for Nusa Kopi 2 and Nusa Kopi 1, respectively.

Table 4. Alternatives in Scenario 2

Milk coffee from	Time of day	Prices
Nusa Kopi 2	Morning	Run for all possible price levels
Nusa Kopi 1	Rest of the day	Run for all possible price levels
Filosofi Kopi	Rest of the day	Rp22,000
Kopi Tuku	Rest of the day	Rp15,000
Kopi Kalyan	Rest of the day	Rp15,000
Anomali Coffee	Rest of the day	Rp22,000

$$\hat{s}_1(p_1) = \begin{cases} 0.12 + 2.23 \cdot 10^{-5} p_1 - 2.39 \cdot 10^{-9} p_1^2 + 7.96 \cdot 10^{-14} p_1^3, & 10000 \leq p_1 < 15000 \\ 1.05 - 1.64 \cdot 10^{-4} p_1 + 1.00 \cdot 10^{-8} p_1^2 - 1.96 \cdot 10^{-13} p_1^3, & 15000 \leq p_1 < 22000 \\ -3.43 + 4.48 \cdot 10^{-4} p_1 - 1.78 \cdot 10^{-8} p_1^2 + 2.25 \cdot 10^{-13} p_1^3, & 22000 \leq p_1 < 28000 \\ 2.70 - 2.09 \cdot 10^{-4} p_1 + 5.67 \cdot 10^{-9} p_1^2 + 5.40 \cdot 10^{-14} p_1^3, & 28000 \leq p_1 < 35000 \\ 0, & otherwise \end{cases} \tag{10}$$

$$\hat{s}_2(p_2) = \begin{cases} 0.13 + 2.64 \cdot 10^{-5} p_2 - 3.01 \cdot 10^{-9} p_2^2 + 1.00 \cdot 10^{-13} p_2^3, & 10000 \leq p_2 < 15000 \\ 1.21 - 1.89 \cdot 10^{-4} p_2 + 1.14 \cdot 10^{-8} p_2^2 - 2.19 \cdot 10^{-13} p_2^3, & 15000 \leq p_2 < 22000 \\ -3.64 + 4.72 \cdot 10^{-4} p_2 - 1.87 \cdot 10^{-8} p_2^2 + 2.37 \cdot 10^{-13} p_2^3, & 22000 \leq p_2 < 28000 \\ 2.77 - 2.15 \cdot 10^{-4} p_2 + 5.84 \cdot 10^{-9} p_2^2 - 5.56 \cdot 10^{-14} p_2^3, & 28000 \leq p_2 < 35000 \\ 0, & otherwise \end{cases} \tag{11}$$

Using the same assumption of D and c, we came up with optimal prices of Rp15,000 for Nusa Kopi 2 and Rp23,000 for Nusa Kopi 1 with total contribution of Rp79,823,200. This is an 11% increase compared to estimated total contribution with one-fare-class under current pricing of Rp71,907,200.

4. Conclusion

We developed a model for determining optimal prices in uncapacitated multi-fare-class problem of a coffee shop, where demand for different fare-classes are interdependent. The representation of this interdependence is rooted in consumers' preferences, which are quantified using utility values. By combining Bayesian estimation, randomized first choice simulation, and cubic spline interpolation, continuous demand functions are derived from choice data. This preference-based demand model results in an optimization problem where the parameters of the objective function change with the value of decision variables.

We implemented the model in a coffee shop in Jakarta, Indonesia, under a simplified two fare-class pricing scenario where the possible values of one fare-class are restricted to price levels set in the DCE questionnaire. Using enumeration, we came up with optimal prices of Rp15,000 for lower-price class and Rp23,000 for higher-price class with total contribution of Rp79,823,200, an 11% increase compared to estimated total contribution with one-fare-class under current pricing of Rp71,907,200.

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