AGGREGATE PRODUCTION PLANNING OF WOODEN TOYS USING MODIFIED PARTICLE SWARM OPTIMIZATION

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ABSTRACT
Aggregate Production Planning was done on small company that produces wooden toys categorized into two types: X and Y. The APP was formulated as an integer linear programming model with an objective function of minimizing production cost. It was optimized using Particle Swarm Optimization (PSO). Four conclusion can be made. First, the number of workers is held constant without hiring or firing in the first month. Second, no overtime from the first month to the last. Third, production level of product X and Y is made exactly the same as the result of forecasting. Lastly, no inventory in the end of the each month.
Key words: Aggregate Production Planning, Integer linear Programming, Particle Swarm Optimization, wooden toys.

1. INTRODUCTION

1.1. Background
Economic growth of Indonesia according to BPS (2015) was approximately 4.71% that also increase the number of business practitioner in the Small to Medium Enterprises (SMEs). SMEs of toys is a part of the SME in Indonesia. However, SMEs that produces toys is now competing really hard with the import toys products. These SMEs need more competitive advantage in able to survive. Wooden toys industry will be the focus of the research. Within the wooden toys industry in Indonesia, there is a serious problem. One of the problem is the difficulty to protect the intellectual property of the company. Making the competition not only about the quality of the product, but really about cost efficiencies of the competitor. So it is needed for SMEs that is related to the wooden toys industry to increase its competitive advantage by making it cost effective and efficient. To increase the effectivity and the efficiencies of these SMEs.

APP was used in this research to minimize the production cost in a company named CV. MK. Now, CV MK still use intuition when planning their production activity which leads to accumulation of inventory level. Which also lead to high costs.

2. THEORETICAL BACKGROUND

2.1. Aggregate Production Planning
Aggregate Production Planning (APP) is a way of company to adjust the level of production to demand in the planning horizon. APP is a high level planning and a difficult task (Chakrabortty, et al 2015). APP also plan the resources including the number of production workers, production level, inventory level, subcontracted product, and backlog in a planning horizon. APP also have a capacity constraint (Al-e, et al, 2012; Wang dan Yeh, 2014).

In this research, APP is performed to plan the level of production, number of workers, and inventory level with minimum cost to fulfill demand.

2.2. Integer Linear Programming for APP
To solve APP cases, integer linear programming (ILP) can be used to minimize the cost in production operation in the planning horizon. The first step in APP is to identify the demand that must be satisfied. Then, identify the cost that is related to production cost such as the labour cost and production cost. After that, determine the
variable decision and the parameter that constrain the decision variable.

This is the typical formulation of APP case using integer linear programming according to Wang and Yeh (2014) and Chakraborrty (2015).

Sets and Indexes

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{ped}$</td>
<td>Total Period in planning horizon</td>
</tr>
<tr>
<td>$n_{wkr}$</td>
<td>Number of worker’s type</td>
</tr>
<tr>
<td>$t$</td>
<td>Period index</td>
</tr>
<tr>
<td>$j$</td>
<td>Index for worker’s type</td>
</tr>
</tbody>
</table>

Parameter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{prd}$</td>
<td>Material cost for each product unit</td>
</tr>
<tr>
<td>$c_{inv}$</td>
<td>Inventory cost each product unit</td>
</tr>
<tr>
<td>$c_{blg}$</td>
<td>Backlog cost each unit product</td>
</tr>
<tr>
<td>$c_{sct}$</td>
<td>Subcontract cost each unit product</td>
</tr>
<tr>
<td>$c_{jhir}$</td>
<td>Cost of recruiting and training for j types of worker</td>
</tr>
<tr>
<td>$c_{jlof}$</td>
<td>Cost of firing for j types of worker</td>
</tr>
<tr>
<td>$c_{jot}$</td>
<td>Overtime cost for worker type j</td>
</tr>
<tr>
<td>$h_{jprd}$</td>
<td>Workhours needed to produce product type j</td>
</tr>
<tr>
<td>$h_{jrt}$</td>
<td>Workhours of worker type j each period</td>
</tr>
<tr>
<td>$h_{ot}$</td>
<td>Overtime capacity for each period</td>
</tr>
<tr>
<td>$d_{t}$</td>
<td>Demand that is needed to be fulfilled in period t</td>
</tr>
<tr>
<td>$U_{t}$</td>
<td>Usage of machinehours each product</td>
</tr>
<tr>
<td>$M_{t}$</td>
<td>Capacity of machine hours</td>
</tr>
</tbody>
</table>

Decision variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{jt}$</td>
<td>Workforce level type j on period t</td>
</tr>
<tr>
<td>$W_{jhir}$</td>
<td>Number of workforce type j that is recruited on period t</td>
</tr>
<tr>
<td>$W_{jlof}$</td>
<td>Number of workforce type j that is fired on period t</td>
</tr>
<tr>
<td>$H_{jot}$</td>
<td>Overtime workhours of worker type j each period</td>
</tr>
<tr>
<td>$P_{tprd}$</td>
<td>Number of product that is produced on period t</td>
</tr>
<tr>
<td>$P_{tinv}$</td>
<td>Inventory level of product on period t</td>
</tr>
<tr>
<td>$P_{tblg}$</td>
<td>Number of product that is kepted during the end of period t</td>
</tr>
<tr>
<td>$P_{tsct}$</td>
<td>Number of product that is subcontracted on period t</td>
</tr>
</tbody>
</table>

2.3. Particle Swarm Optimization for APP

Particle Swarm Optimization (PSO) is a metaheuristic algorithm introduced by Eberhart and Kennedy (1995). This metaheuristic optimization algorithm is inspired by the movement of a flock of birds or a school of fish when trying to survive using their memory and their communication.

PSO algorithm is applied in this research as follows: First, it raises a population that consist of possible solution. Each possible solution or particle has a position $X_i = (x_1, x_2, ..., x_d)$ and a velocity $V_i = (v_1, v_2, ..., v_d)$. Each particle has a memory of their solution (Pbest) and in a swarm there is one best possible solution (Gbest). Each particle also have fitness value which represent how good a solution is. in this case, the fitness value will be the APP function. Then using equation 1 and 2, $X_i$ and $V_i$ will be updated each iteration until it reaches the predetermined maximum iteration ($T_{max}$). The number of maximum iterations in PSO can be declared valid if the results of PSO answer converges to a certain number and does not change if the population or iteration is added. PSO standard algorithm flowchart is shown in Figure 1.

\[ V_{id}^{new} = W V_{id}^{old} + c_1 \cdot \text{rand1} \cdot (P_{bestid} - X_{id}) + c_1 \cdot \text{rand2} \cdot (G_{bestid} - X_{id}) \]  

\[ X_{id}^{new} = X_{id}^{old} + V_{id}^{new} \]  

Figure 1. Flow Chart of PSO Algorithm  
(Eberhart and Kennedy, 1995)

In the PSO algorithm Kuo and Yang (2011), the value of $c_1$ and $c_2$ in formula 1 are a value of learning factor, while $\text{rand1}$ and $\text{rand2}$ are random numbers.
and rand2 are two random numbers on the interval [0, 1].

In this research the W value in formula 1 is described as an inertial value which will fasten convergence speed (Shi and Eberhart, 1999). The calculation W value for each iteration is shown in equation 3.

\[
W(t) = W_{\text{max}} - (W_{\text{max}} - W_{\text{min}}) \frac{t}{T_{\text{max}}}
\]  

(3)

Wmax and Wmin are 0.9 and 0.4 respectively which is recommended.

### 2.4 Modified Particle Swarm Optimization

The writer applied the Modified Particle Swarm Optimization (MPSO) Wang and Yeh (2014) to the problem of APP by adding aspects of sub-swarm to the algorithm. Each period/row represent a sub-swarm. Therefore, in this case Vi and Xi are updated by a random number generator each period. MPSO algorithm flowchart is shown in Figure 2.

![Flowchart of MPSO Algorithm](Wang and Yeh, 2014)

### 2.5 Penalty Function

Metaheuristic algorithm was created to solve unconstrained problem. Because APP is a constrained problem, APP must be converted to a unconstrained problem. Because APP objective function in this research is to minimize the production cost in general, then the penalty function was made to penalize a possible solution that violate the APP constraint by adding the objective function value. Penalty function shown in Equation 4.

\[
P(t) = \sum_i (r_i \cdot \max[0, G_i(x)])^2 + \sum_j (c_j[H_j(x)])^2
\]  

(4)

r_i and c_j is a large positive value to worsen the solution that violate the mathematical constraint. Value G_i(x) is a mathematical constrain in the form of inequality that has been in the balance to the left side of “≤” sign. While the value of H_j(x) is a mathematical constrain in the form of the equation in balance to one side of “=”. 

### 3. RESEARCH METHOD

The method begins by identifying the problem, followed by the formulation of the problem, followed by setting goals, followed by conducting a literature study, followed by the collection of data, followed by data processing, followed by analysis, and ends with conclusions and suggestions.

In problem identification step, observation will be made in the form of interviews with the company. The output of this step is founding of problem definition issues to be addressed in this study. The motive of this research is to make CV. MK able to perform aggregate planning more efficient production so as to increase competitiveness.

As a continuation of problem identification, problem formulation made from the CV. MK. The formulation of the problem in this research is what are the costs involved, and how to determine the production plan according to demand at a minimal cost.

The next step is to conduct a literature study based on previous studies on the issue of APP. One of the main reference is from Wang and Yeh (2014) titled Modified Particle Swarm Optimization for Aggregate Production Planning”.

In the data collection phase, the information required as input APP CV. MK sought by interview and using some assumption. Primary data that will be needed include wooden toys product sales history, company policies in adjusting
production levels to demand, the costs involved in APP CV. MK. As for the secondary data, labour law is used to calculate the cost assumptions for hiring, firing, and overtime hours.

In this research data processing starts with selecting the products to be studied using ABC analysis. Then proceed with the forecasting demand in the planned period using the 3 method which is Triple exponential smoothing 1 parameter, double exponential smoothing 2 parameters, and linear regression. After the forecasting process, results with the smallest error value is selected to be the reference APP on the CV. MK.

Next step is to determine the objective function and the mathematical constraints. The constraints must be added to the objective function in a form of penalty function. Then objective function value will be optimized using a modified particle swarm optimization (MPSO). The MPSO algorithm code is created using MATLAB R2015a.

Once the objective function of APP optimization has been done, then the next step is the analysis of the results of APP. Specifies the number of regular workers, overtime, production levels, and optimal inventory levels with the level of production to satisfy the demand. Based on observation, data collection, data processing, and analysis made a conclusion which regular workers, overtime, production levels, and optimal inventory levels with the level of production needed to satisfy demand. After that, conclusion was made.

4. RESULT AND DISCUSSION

4.1 Aggregate Production Planning (APP)

APP is only done for 3 months of planning horizon. APP is needed to adjust the level of production with forecasted demand using the correct amount of resource so it is optimized. There are three way that CV. MK can control the level of production which is:

1. Recruitment of firing production workforce (\(W_{j_t}^{hir}\) or \(W_{j_t}^{lof}\))
2. Adding overtime (\(H_{j_t}^{ot}\))
3. Store inventory from the previous period (\(P_{t}^{inv}\))

In this research, APP objective function and its constraints was formulated as an integer linear programming that fit the APP case in CV. MK. Because CV. MK do not use backlog and subcontract, those variable was deleted. And because this research categorized product as X and Y, there will be production level and inventory level for both X and Y. Objective function and its constraints are shown in equation 5 to 12.

Objective Function:
\[
\text{Min} \sum_i \sum_j c_{ij}^{rt} \cdot h_{j_t}^{rt} + \sum_i \sum_j c_{ij}^{ot} \cdot H_{j_t}^{ot} + \sum_i c_i^{hir} \cdot W_{j_t}^{hir} + c_j^{lof} \cdot W_{j_t}^{lof} + \sum_i \left[ (c^{inv}(X), P_t^{inv}(X)) + \sum_i (c^{inv}(Y), P_t^{inv}(Y)) \right] + \sum_i \left[ (c^{prd}(X), P_t^{prd}(X)) + \sum_i (c^{prd}(Y), P_t^{prd}(Y)) \right]
\]

Constraints:
1. Constraint of workforce
   \[
   W_{j_t} = W_{j_t(t-1)} + W_{j_t}^{hir} - W_{j_t}^{lof}
   \]
2. Constraint of production capacity
   \[
   p_{t(x)}^{prd} \cdot h_{j_t}^{pr} + p_{t(y)}^{prd} \cdot h_{j_t}^{pr} \leq \sum_i h_{j_t}^{rt} \cdot W_{j_t} + H_{j_t}^{ot}
   \]
3. Constraint from inventory level X and Y
   \[
   P_{t(x)}^{inv} + p_{t(x)}^{prd} = d_t(X) + P_{t(x)}^{inv}
   \]
   \[
   P_{t(y)}^{inv} + p_{t(y)}^{prd} = d_t(Y) + P_{t(y)}^{inv}
   \]
4. Constraint of overtime hours
   \[
   H_{j_t}^{ot} \leq h^{ot} \cdot W_{j_t}
   \]
5. Where \(h^{ot} = 2\) hours
6. Constraint of machine capacity
   \[
   \sum_i U_i \cdot (W_{j_t} + h_{j_t}^{rt}) \leq M_i
   \]
   Where \(M_i = 192\) hours
7. Constraint from tumbler machine capacity
   \[
   \sum_i U_i \cdot p_{t(y)}^{prd} \leq M_t(Y)
   \]
   Where \(M_t(Y) = 192\) hours

The constraints above was changed into penalty functions. Then new unconstrained objective function is shown as Equation 13.

\[
\text{Min} \sum_i \sum_j c_{ij}^{rt} \cdot h_{j_t}^{rt} \cdot W_{j_t} + \sum_i \sum_j c_{ij}^{ot} \cdot H_{j_t}^{ot} + \sum_i c_i^{hir} \cdot W_{j_t}^{hir} + c_j^{lof} \cdot W_{j_t}^{lof} + \sum_i \left[ (c^{inv}(X), P_t^{inv}(X)) + \sum_i (c^{inv}(Y), P_t^{inv}(Y)) \right] + \sum_i \left[ (c^{prd}(X), P_t^{prd}(X)) + \sum_i (c^{prd}(Y), P_t^{prd}(Y)) \right]
\]
The best solution from MPSO will be used as the answer to the APP problem. The optimal workforce level, overtime hours, production level, and inventory for each period in the planning horizon is shown in Table 1. The optimal APP solution costs Rp 184.89 million.

Based on Table 1, it can be seen that the CV. MK, does not need to have inventory in the end of the month or the difference between production and demand is equal to zero. Based on the results of the PSO optimization, the production levels is the same as forecasted demand. And because the objective function value already converges to a number with 2000 iterations and 1000 population, the optimization results can be declared as valid. Figure 3 displays the objective function value at each iteration. At about 1200 iteration, the value of the objective function is already convergent and if the populations or iterations added, the value of the objective function will not change.

Table 1. MPSO APP Result

<table>
<thead>
<tr>
<th>Month</th>
<th>Regular workforce hiring</th>
<th>Workforce hiring</th>
<th>Overtime hours</th>
<th>Production (X)</th>
<th>Inventory (X)</th>
<th>Production (Y)</th>
<th>Inventory (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>1202</td>
<td>0</td>
<td>742</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>1201</td>
<td>0</td>
<td>762</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>1200</td>
<td>0</td>
<td>782</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3. Graph of objective value to each iteration
5. CONCLUSION

Based on the results obtained using MPSO, it can be concluded that CV. MK should plan their production using APP instead of intuition. There are four conclusions that can be drawn from this study.

1. The number of production workers in the first month until the third month is 25 people without firing workers in the first month.
2. No use of overtime hours in the first month until the third month.
3. Total production for product X and Y identical to forecasting results. For product types X, the production level numbered 1202, 1201, and 1200 units in month 1, 2, and 3 respectively. As for the type of product Y, the level of production level is 742, 762, and 782 units in month 1, 2, and 3 respectively.
4. There is no inventory in the end of the each month in planning horizon.

APP minimal costs Idenfied from MPSO amounting to Rp 184.89,- million

6. REFERENCES


AUTHOR BIOGRAPHIES

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