# INVENTORY DETERMINATION MODEL FOR PACKAGING MATERIALS WITH VARIOUS DEMAND DATA DISTRIBUTIONS IN CHEMICAL COMPANY

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***ABSTRACT***

*This research discusses about inventory management of packaging materials in chemical company. The company used periodic review model (P model) which cause low level of inventory and high inventory cost due to the not normally distributed demand data. Recently, the company decided to change the inventory control system to be a continuous review model. This esearch is conducted to determine the right reorder point and quantity order for each of the packaging materials as one of the requirement for continuous review model. This esearch also equipped with the analysis of total inventory cost between two types of continuous review models which are simple probabilistic model and Q model that consider three kinds of demand data distribution; normal, uniform, and exponential. Hadley Within iteration method is used to obtain the optimal inventory since the analytical method is hard to solve. After processing the data, it can be concluded that Q model can substitute the P model because it results in lower total inventory cost as much as 25.77% than the simple probabilistic model.*

***Keywords**: Inventory systems, continuous review model, normal, uniform, exponential*

## 1. INTRODUCTION

Inventory is the stock of various types of materials or resources that kept its existence in an organization. It includes the raw material, work-in-process, finished goods, and distribution inventory considered owned by the company. (Chase & Jacobs, 2008).

Inventory control at the desired level in the operational activities of a company, is one of the important activity in order to meet market needs while controlling inventory costs incurred.

One of the largest multinational polycarbonate in Southeast Asia is one of the companies that still having problems in determining inventory decisions. Currently one its four divisions named Functional Films plant has a very fluctuating historical demand data. This division produce the products in the form of thin layers using a polycarbonate base that could be apply to electronics panel, display, automotive, and also id cards. Those products made in Functional Films Division can easily become defect if it is not well packaged. Based on this consideration, it is require the finished products to be wrapped with a specific type of packaging material for different type of products. Each products are packaged in different method, depends on the kind of product and customer request. These differences result in a difference of packaging materials used. At this moment, the division has not found the right inventory decisions to control the inventory level of those various types of packaging materials in the warehouse.

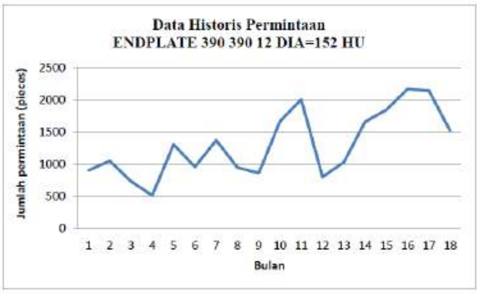


Figure 1. Historical demand data of endplate

Figure 1 imply that the demand from customers is very fluctuates. Some of them even not followed the normal distribution. Consequently, PPIC department meet some difficulties in forecasting and determining

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what quantity to order every time the packaging materials ordering time has come. It is hard to find the right reorder point and quantity order for each type of materials.

Several previous researches had found formula that able to help in determining the proper inventory system as the basic of inventory control at the company.

Li et al. investigated reorder point optimization strategy of assembly manufacturing system with random demand and random lead time. The demand itself is assumed to follow normal distribution. They are searching for the best manufacturer order strategy to gain minimum integration of the supply chain inventory cost (Li, Zhan, & Jin, 2013).

Another research seeking for optimum way to minimize annual inventory cost with determining production lot size, reorder point, and investment to reduce setup cost. Wang et al. consider the historical demand data that follow uniform and exponential distribution (Wang, Shu, &Chao, 2006).

Finally, El-Wakeel Mona F. examined the probabilistic backorders inventory system when the order cost unit is a function of the order quantity. The objectives of this paper is to minimize the expected annual total cost when the lead time demand follows the uniform distribution (F. El-Wakeel, 2011).

Table 1 display the state of art of this research.

Table 1 State of art

| Paper / Model Component | Li, Zhan, & Jin (2013) | Wang, Shu, & Chao (2006) | Mona F.El-Wakeel (2011) | Inaki M.Hakim & Putri Larassati (2014) |
|---|---|---|---|---|
| **Performance Criteria** | Minimizing inventory total cost for the entire supply chain | Minimizing annual inventory cost | Minimizing annual inventory cost | Minimizing inventory cost |
| **Decision Variables** | Reorder point ( $r$ ) and Quantity order ($Q$) | Lot size ($Q$) and Reorder point ( $R$ ) | Reorder point ($R$) and Quantity order (Q) | Reorder point ($r$) and Quantity order ($q_o$) |
| **Demand Data Distribution** | Normal | Uniform, Exponential | Uniform | Normal, Exponential, Uniform |

## 2. THEORETICAL BACKGROUND

### 2.1 Types of inventory

According to Nur Bahagia (2006), inventory can be classified into three types:

a. *raw material*
b. *work in process*
c. *finished goods*

Those kinds of inventory are explained further below:

1. Raw material. Raw material is earliest material input before the production process started.
2. Work in process. This type of inventory is the transition from raw material to finished product. Their existence cannot be avoided in the job order production process since the long time needed to transform raw materials into finished goods.
3. Finished goods. Finished goods are the final result of the whole production process and will be delivered to the end customers

### 2.2 Component of inventory cost

There are many things calculated in inventory total cost formula. The inventory total cost is consists of several types of charge as follows (Nur Bahagia, 2006):

1. Purchasing cost
   This cost must be incurred to purchase material from supplier.
1. Procurement cost
   This cost issued when the procurement activity is held. Procurement cost consists of ordering cost (when material comes from the outside) and setup cost (when materials come from inside company).
3. Holding cost
   Holding cost is charged for maintaining warehouse facilities, inventory handling activity, insurance, depreciation, tax, and capital opportunity cost.
4. Shortage cost
   Shortage cost exists when company cannot fulfill customer demand. If the company suffered material shortage, there will be two possibilities. Their customers will be waiting for the material until they get it or cancel their order and find another company that able to serve them as they request. Lost customers as material shortage consequences can be defined as shortage cost.

### 2.3 Probabilistic inventory systems

An inventory system provides the organizational structure and the operating policies for maintaining and controlling goods to be stocked. It also encompasses activities and decisions to control material stock. This system classified into two types as follows:

1. A single-period inventory model
   This kind of inventory system is useful for determining inventory decision to anticipate single-period demand. It also known as newsboy model and being introduced for the first time by Within (1955).
2. Multi-period inventory systems
   There are two general types of multi-period inventory systems: Fixed-order quantity models (Q-model) and Fixed-time period models (P-model). Table 2 explains the differences between those two models.

Table 2. Q-model and P-model differences

| Feature | Q-Model | P-Model |
|---|---|---|
| Order quantity | Same amount ordered each time | Varies each time order is placed |
| When to order | When inventory position drops to the reorder level | When the review period arrives |
| Size of inventory | Less than p-model | Larger than q-model |
| Record-keeping | Each time a withdrawal or addition is made | Only at review period |
| Time to maintain | Higher due to peIDRetual recordkeeping | |
| Type of items | Higher-priced, critical, or important items | |

Source: Operations and Supply Chain Management (The Core) by Jacobs, Robert and Chase, Richard (2008)

### 2.4 Probability density function characteristics

Historical demand data are fluctuates and follow normal, exponential, and uniform distribution. Harinaldi (2005) wrote their characteristics as follows:

1. Normal distribution
   Probability density function of this kind of distribution with mean $\mu$ and variance $\sigma^2$ is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad (1)$$

2. Exponential distribution
   Exponential distribution is one kind of gamma distribution with $\alpha = 1$ and $\beta = 1/\lambda$. (Navidi, 2006). If continuous random variable X follows exponential distribution and having $\lambda$ as parameter, with $\lambda > 0$, probability density function of X is:

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (2)$$

3. Uniform distribution
   This kind of distribution consists of several random variable X which has a finite value at a certain interval [*a*,*b*]. Uniform probability density function can be defined as follows:

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

## 3. RESEARCH METHOD

This research determining inventory decisions using inventory probabilistic system with Q model. Hadley-Within iteration is used to solve the equation because it is hard to find the solution analytically (Taha, 2011)

**Performance Criteria**

Our objective is to minimize the annual total inventory cost with the following equation:

Minimizing $O_T = O_b + O_p + O_s + O_k$ (4)

Minimizing $O_T = Dp + \frac{aD}{q_o} + h\left(\frac{1}{2}q_o + r - D_L\right) + c_u \frac{D}{q_o}\int_r^{\infty}(x-r)f(x)dx$ (5)

**Parameters**

Based on performance criteria, these are parameters used in the model:

$p$ = Purchasing cost per unit
$a$ = Ordering cost for each order activity
$h$ = Holding cost per unit per month
$c_u$ = Shortage cost per unit

**Decision Variables**

There are two decision variables related to the model. These variables also used for determining inventory decision.

$q_o$ = quantity order lot size for each ordering activity

*r* = Inventory level when the order is placed (reorder point)

**Inventory Decisions**

The optimal value of reorder point and quantity order which minimize Eq. (5) can be found by setting each of the corresponding first partial derivatives of Eq. (5) to zero. From this calculation, we obtain:

$$q_o = \sqrt{\frac{2D[a + c_u \int_r^{\infty}(x-r)f(x)dx]}{h}} \qquad (6)$$

$$\int_r^{\infty} f(x)dx = \frac{hq_o}{c_u D} \qquad (7)$$

According to Eq. (6) and Eq. (7), these are reorder point and quantity order formulation for each kind of demand data distribution.

**Normal distribution**

$$q_o = \sqrt{\frac{2D[a + c_u\,(S_L[f(z_\alpha) - z_\alpha \psi(z_\alpha)])]}{h}} \qquad (8)$$

$$r = DL + z_\alpha S\sqrt{L} \qquad (9)$$

**Exponential distribution**

$$q_o = \sqrt{\frac{2D(a + c_u\,(\lambda e^{-\lambda r}))}{h}} \qquad (10)$$

$$r = \frac{\ln(\frac{q_o}{k})}{-\lambda} \qquad (11)$$

*k* is constant and its value is based on parameter $\lambda$ from historical data of demand.

**Uniform distribution**

$$q_o = \sqrt{\frac{2D(a + c_u \int_r^{\infty}(x-r)\frac{1}{b-a}dx)}{h}} \qquad (12)$$

$$\int_r^{\infty} \frac{1}{b-a}dx = \frac{hq_o}{c_u D} \qquad (13)$$

## 4. RESULT AND DISCUSSION

### 4.1 Inventory decisions

Table 3 summarized each material inventory decisions with different kind of distribution approach.

Each of those inventory decisions are compared with inventory decisions resulted by simple inventory probabilistic model. Analysis is conducted to know the differences of reorder point, quantity order, and inventory total cost between those two different models.

Table 3. Reorder point and quantity order for each material

| Material article no. | Normal distribution | | Exponential distribution | | Uniform distribution | |
|---|---|---|---|---|---|---|
| | r | $q_o$ | r | $q_o$ | r | $q_o$ |
| 80236647 | 171 | 446 | x | x | x | x |
| 82037625 | 78 | 237 | x | x | 429 | 112 |
| 80369485 | 44 | 133 | x | x | x | x |
| 80369396 | 551 | 679 | x | x | 89 | 3785 |
| 81549681 | 336 | 524 | x | x | 1808 | 4532 |
| 80236442 | 136 | 275 | x | x | x | x |
| 82528742 | 70 | 152 | 264 | 268 | x | x |
| 80719397 | 8 | 64 | 92 | 106 | x | x |
| 80660333 | 13 | 52 | x | x | x | x |
| 81458856 | x | x | 1470 | 1497 | x | x |
| 81349712 | x | x | 753 | 904 | x | x |
| 82525875 | x | x | 275 | 287 | x | x |
| 83777010 | x | x | 70 | 90 | x | x |
| 80669551 | x | x | 27 | 42 | x | x |

### 4.2 Analysis of reorder point

Figure 2, 3, and 4 show the differences of reorder point in Q-model and simple probabilistic inventory model. *r** represents reorder point value with simple probabilistic model whereas *r* is reorder point for Q-model.

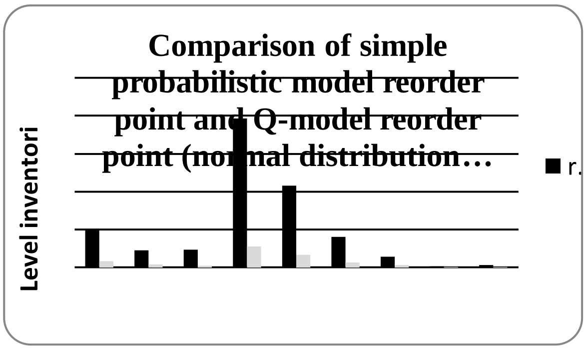


Figure 2. Diagram of reorder point comparison between simple probabilistic model and Q-model (normal approach)

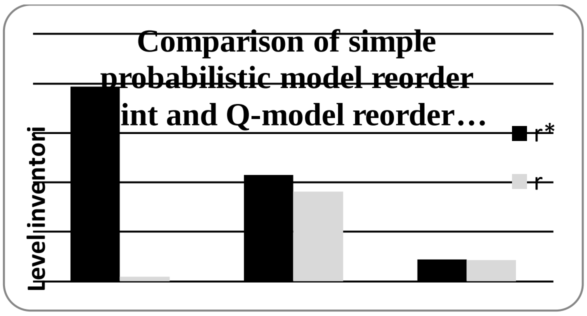


Figure 3. Diagram of reorder point comparison between simple probabilistic model and Q-model (uniform approach)

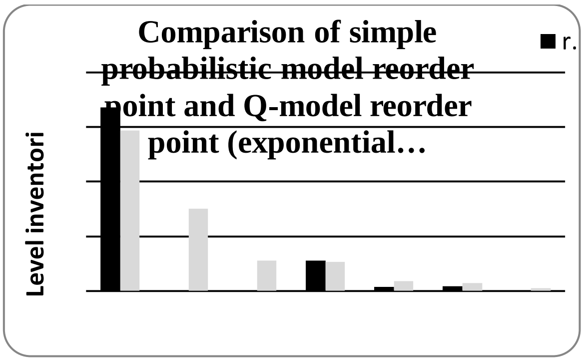


Figure 4. Diagram of reorder point comparison between simple probabilistic model and Q-model (exponential approach)

Generally, reorder point of Q-model is smaller than the reorder point of simple probabilistic model. It is showed that ordering time for Q-model will be longer than simple probabilistic model.

### 4.3 Analysis of quantity order

Analysis of quantity order differences between Q-model and simple probabilistic model are shown in these figures. *q** represents quantity order using simple probabilistic model (with *service level* = 95%) whereas $q_o$ shows the Q-model *quantity order*.

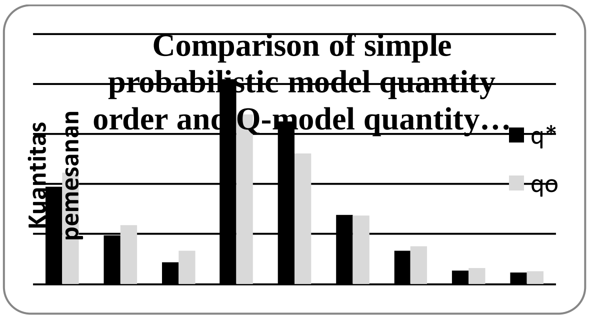


Figure 5. Diagram of quantity order comparison between simple probabilistic model and Q-model (normal approach)

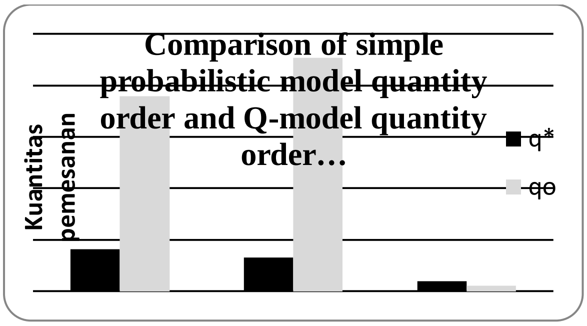


Figure 6. Diagram of quantity order comparison between simple probabilistic model and Q-model (uniform approach)

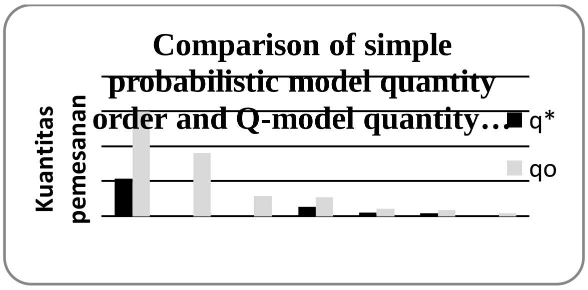


Figure 7. Diagram of quantity order comparison between simple probabilistic model and Q-model (exponential approach)

According to those three diagrams, simple probabilistic model quantity order is smaller than Q-model quantity order. When Q-model is used, the company will order with larger lot size than simple probabilistic model to reduce order cost.

### 4.4 Total inventory cost analysis

Table 4 illustrates and compares total inventory cost with Q-model and simple probabilistic model.

Table 4. Comparison of annual inventory cost simple probabilistic model and Q-model.

| Material Article No. | Simple probabilistic model cost | Q-model cost |
|---|---|---|
| 80236647 | IDR12,102,099.31 | IDR9,754,995.52 |
| 82037625 | IDR3,476,100.61 | IDR2,831,953.59 |
| 80369485 | IDR3,217,811.13 | IDR1,694,340.85 |
| 80369396 | IDR53,863,399.54 | IDR36,261,589.35 |
| 81549681 | IDR32,800,805.04 | IDR25,033,709.92 |
| 80236442 | IDR5,697,579.32 | IDR4,351,312.12 |
| 82528742 | IDR2,830,438.32 | IDR2,271,638.16 |
| 80719397 | IDR2,502,738.17 | IDR2,366,225.00 |
| 80660333 | IDR513,096.95 | IDR413,358.97 |
| 81458856 | IDR1,956,001.12 | IDR2,256,499.37 |
| 83777010 | IDR1,675,519.46 | IDR2,308,184.10 |
| TOTAL | IDR120,635,588.96 | IDR89,543,806.95 |
| **Difference** | **25.77%** | |

Table 4 imply that Q-model with data distribution consideration is the proper inventory control model since this model reduces up to 25.77% inventory cost compare with simple probabilistic model. .

### 4.5 Sensitivity analysis

Sensitivity analysis is conducted to identify the most influential parameter affected inventory decisions. It is done by changing parameters in the model 30% higher and lower than before.

Sensitivity ratio then calculated to determine parameter change that most affected inventory decisions. This is the ratio of the change in model output per unit change in variable input. Table 5 describe sensitivity ratio for each parameter related to reorder point change. $X_1$ as the representative of total cost before any changes and $X_2$ is total cost after 30% change. On the other hand, $Y_1$ is the inventory decision before any parameter change and $Y_2$ is the inventory decisions resulted after 30% change on parameter.

Table 5. Sensitivity ratio calculation related to reorder point value

| Parameter | $X_1$ | $X_2$ | $Y_1$ | $Y_2$ | SR |
|---|---|---|---|---|---|
| Purchase cost | 25000 | 20000 | 551 | 551 | 0 |
| Order cost | 125000 | 87500 | 551 | 552 | 0.006 |
| Holding cost | 5000 | 3500 | 551 | 574 | 0.139 |
| Shortage cost | 100000 | 70000 | 551 | 528 | 0.139 |

Table 5 shows that shortage cost as well as holding cost are the most influential parameters on reorder point value.

Table 6. Sensitivity ratio calculation related to quantity order value

| Parameter | $X_1$ | $X_2$ | $Y_1$ | $Y_2$ | SR |
|---|---|---|---|---|---|
| Purchase cost | 25000 | 30000 | 679 | 679 | 0 |
| Order cost | 125000 | 162500 | 679 | 693 | 0.07 |
| Holding cost | 5000 | 6500 | 679 | 596 | 0.41 |
| Shortage cost | 100000 | 130000 | 679 | 719 | 0.2 |

Based on Table 6, holding cost is the most influential parameter affecting quantity order decision.

## 5. CONCLUSION

In this paper, we consider inventory decisions by applying Q-model for each packaging materials used in a chemical company. By assuming the model with backorder condition, it is obtain that inventory decision by using Q-model resulted in lower inventory cost than simple probabilistic model. In the other words, Q-model is the most appropriate inventory control model to replace P-model due to its lower inventory cost and higher inventory availability.

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