MULTI-LEVEL INVENTORY MANAGEMENT CONSIDERING TRANSPORTATION COST AND QUANTITY DISCOUNT

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ABSTRACT

In recent years, Economic Order Quantity model has been studied by many researchers. Some of the researchers extended the model to consider real systems. In this research, we adopt EOQ model considering transportation cost and discount price. We studied the effect of all quantities discount price offered by warehouse on the total cost of retailers and warehouse. Numerical study shows that by 1-3% all quantities discount price with minimum order of 200-600 units, there is 1.37% margin cost. Managerial insights are suggested on how certain parameters influence decision policy.

***Keywords**: Inventory Management, Transportation Cost, All Quantities Discount.*

1. INTRODUCTION

The traditional Economic Order Quantity (EOQ) model captures only the trade-off between inventory carrying cost and setup cost. The transportation cost is usually neglected or included in the setup cost Madadi, Kurz, and Ashayeri (2010).

According to Swenseth and Godfrey (2002), more than 50% of the total annual logistics cost of a product can be attributed to transportation. They suggest that any consideration of purchase quantities should consider transportation costs.

Multi-level Inventory management for single item is one of the most common in basic problem. A lot of studies have been doned on this area. Many researchers had extended this basic model with several assumptions and different environments. One of them is quantity discount price.

As an incentive for a buyer to purchase larger lot size, it is a common practice for the supplier to offer lower unit price for larger order quantities. Suppliers can reduce their set up cost/order cost per unit, increase their sales and liquidate inventory. The buyer benefits are reduced per unit ordering cost and lower unit price. However, it will increase carrying cost by having to hold more inventory (Tersine, 1994).

In quantity discount, customer can purchase products at the lower price when the ordering quantity is over a certain amount (Lee, Kang, Lai, & Hong, 2013). Quantity discount price can either all unit discount or incremental discount quantity. All quantity discounts are given to buyer for the entire lot size when they buy, while in incremental quantity discount, the lower price only applies to unit purchase above a specified quantities.

In this paper, a single item inventory management model is considered with transportation cost and all unit discount offered by the warehouse to the retailers. We investigated the effect of quantity discount to warehouse cost and retailers cost using a numerical example. The model developed by Madadi et. al. (2010) is modified to fit our study. We also do some numerical analysis and compare with some of the real case faced by some warehouses or retailers.

2. THEORETICAL BACKGROUND

2.1 Previous Research

Several studies have been proposed to extend EOQ model to different conditions. Some researchers established models to minimize Inventory cost. Madadi et al. (2010) considered transportation cost to determine EOQ and review period in multi-level inventory for single product. They also developed two models using centralized and decentralized ordering model.

Ritha and Jeyakumari (2011) considered decentralized model with transportation cost consideration in fuzzy environment. They introduced fuzzy parameters and fuzzy replenishment order quantity. The algorithm used the Lagrangian method to solve inequality constrains in fuzzy decentralized ordering model.

Zhao et al.(2004) addressed the optimal ordering quantity and frequency problem for a supplier–retailer logistic system; the transportation cost as well as the multiple vehicles are considered.

In the area of quantity discount, some research have been carried out to find optimum EOQ which minimum Total Inventory Cost. Cárdenas-Barrón (2012) presented the close form for determining the number of shipments for two new inventory model considering items with imperfect quality, quantity discounts, and different holding costs for good and defective items. He provided simple formulae to determine the integral value of the discrete variable without inequalities.

Lee et al. (2013) constructed a mixed integer programming (MIP) model to solve the lot-sizing problem with multiple suppliers, multiple periods and quantity discounts. They proposed Genetic Algorithm (GA) to solve the complicated problem. Their objective is to minimize total costs (ordering cost, holding cost, purchase cost and transportation cost).

2.2 Propose Study

This study aims to obtain minimal total cost in retailer and warehouse considering transportation cost and all quantity discount price in multi-level environment consisting of Supplier-Warehouse-retailers. The costs include ordering cost, holding cost, purchase cost and transportation cost. We use Madadi's algorithm for decentralized ordering model. Decision variable for each retailer is to find the economic order quantity (EOQ) while decision variable for warehouse is to find the review point or the Economic Order Interval (EOI).

From Madadi. et. al.(2010), the formulas are:

- Objective function: Minimizing the total cost
- Each retailers and warehouse have their own strategy to minimize their total cost.

$$Z^c = \sum_{j=1}^{N} [TC_j\,(Q_j)] + TC_W$$

Notation:

N : Number of retailers
J : Retailer index (j = 1,2,3….N)
TC_W : Total Cost Warehouse
Q_j : Order quantity in unit for retailer j

3. RESEARCH METHOD

3.1. Minimize Total Cost on Retailers

Base on Madadi's models, we developed model to find optimum total cost for all quantities discount offered by supplier.

$$C_{R_j} = A_j \frac{D_j}{Q_j} \quad ; \quad C_{C_{jk}} = \left(\frac{Q_j}{2} + K_j.\sigma_{jL_{wj}}\right) P_{jk} r_j \quad ; C_{T_j} = \left(\alpha_j + t_w \left[\frac{Q_j}{VC}\right] d_{wj}\right) \frac{D_j}{Q_j}$$

$$C_{P_j} = P_{jk} D_{jk}$$

$$TC_j(Q_j) = P_{jk} D_j + A_j \frac{D_j}{Q_j} + \left(\frac{Q_j}{2} + K_j.\sigma_{jL_{wj}}\right) P_{jk} r_j + \left(\alpha_j + t_w \left[\frac{Q_j}{VC}\right] d_{wj}\right) \frac{D_j}{Q_j}$$

The number of truck $g_j = \left[\frac{Q_j}{VC}\right]$ and g_j should be positive integer number.

The charts from the above equations are illustrated as follows:

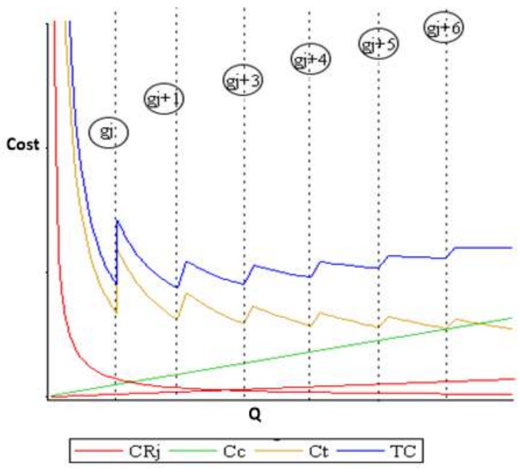

Figure 1 Retailers Total Cost (without purchase cost)

Since the total cost (TC) for whole period is not continuous function, it is not possible to differentiate TC directly.

However, TC function for each number of truck is continuous, thus each total cost for the number of truck used (TC_{gj}) can be differentiated to obtain the minimum total cost. Maple 13 computer software was used to solve the problem.

3.2 Minimize Total Cost of Warehouse

The equations development from Madadi's equation are shown below.

$$C_{R_w} = \frac{A_w}{R_w}\ ;\ C_{S_w} = \frac{B_w}{R_w} Pr_u \geq K_w;\ \ C_{P_w} = P_w D_w$$

$$C_{C_w} = \left(\frac{\mu_{w(R_w+L_{sw})}}{2} + K_w.\sigma_{w(R_w+L_{sw})}\right) P_w r_w$$

$$C_{T_w} = \left(\alpha_s + t_s \left[\frac{\mu_w R_w}{VC}\right] d_{sw}\right) \frac{1}{R_w}$$

z = number of trucks per review = $\left[\frac{\mu_w R_w}{VC}\right]$

$$TC_w(R_w) = P_w D_w + \frac{A_w}{R_w} + \left(\frac{\mu_{w(R_w+L_{sw})}}{2} + K_w.\sigma_{w(R_w+L_{sw})}\right) P_w r_w + \frac{B_w}{R_w} Pr_u + \left(\alpha_s + t_s \left[\frac{\mu_w R_w}{VC}\right] d_{sw}\right) \frac{1}{R_w}$$

Notations for equation above are:

j : retailer index
k : price index
A_j, A_w : Ordering cost in retailer j, warehouse w
D_j, D_w : Demand in retailer j, warehouse w
K_j K_w,: Safety factor in retailer j, warehouse w
$\sigma_{jL_{wj}}$: Variation of Lead Time Demand in retailers
$\sigma_{w(R_w+L_{sw})}$: Variation demand during Review period and Lead Time supplier to Warehouse.
P_{jk}, P_w : Purchase cost per unit in retailer j, warehouse w
r_j, r_w : Fraction of holding cost per unit price in retailer j, warehouse w
α_w, α_s: Fixed transportation cost per shipment from warehouse to retailer j, supplier to warehouse
t_w, t_s : Variable transportation cost from warehouse to retailer, supplier to warehouse.
d_{wj}, d_{sw}: Distance from retailer j to warehouse, warehouse to supplier.
VC : Vehicle Capacity
R_w : Review Period
B_w : Cost per stock out

Figure 2 illustrates the equations.

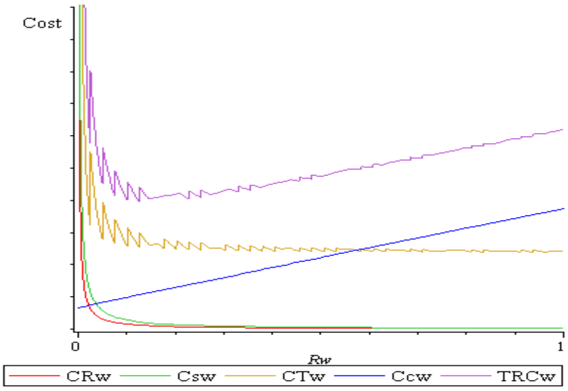

Figure 2 Warehouse Total Cost (without purchase cost)

In this study, some algorithm were modified to calculate total inventory cost at a given price. For each retailer, warehouse will offer discount price (P_{jk}) at minimum quantity order U_{jk}. This quantity discount minimized total cost of inventory in retailer. Retailer decided optimum lot size at related price which minimize total inventory cost. Large lot size reduced purchase cost and ordering cost while holding cost increased. Retailers captured the tradeoff between ordering cost, purchase cost and holding cost.

3.3 Solution methods

Some assumptions are used for modeling:

1. Demands for retailers are stochastic and follow a normal distribution.
2. Safety stock is used to anticipate demand fluctuation.
3. Single item inventory system.
4. Lead Time is known and fixed.
5. Each retailer uses a continuous review policy.
6. Lost sales are not considered.
7. Warehouse uses a periodic-review policy.
8. All units discount are offered to retailers.
9. Seller applied *non-free on board* selling to its customer.

3.3.1 For retailer

Retailers apply EOQ and continuous review policy.

Algorithm:

1. Start from the lowest price
2. Set price P_{jk} for U_{jk} and U_{jk-1}.
3. Compare U_{jk} with Vehicle Capacity(VC).

4. $U_{jk}/_{VC} = a$. Set "a" as integer point by round up the number $g_j^a = g_j$
5. Find Q_j^* as initial number of truck.
6. Calculate TC_{g_j}
7. Go to step 8 for Q_j^* at g_{j+1}
8. Calculate $TC_{g_{j+1}}$
9. Continue step 5 to step 8 until minimum total cost (TC) is obtained and $Q_j^* \leq \mu_j$
10. Do the same for each price break and the Q with the least total cost is the optimal value.
11. End

3.3.2. For Warehouse

In warehouse, Periodic Review has been used considering the Optimal Review Point as a decision variable in minimizing the total cost. Therefore, warehouse has to fulfill all retailers demand during a period time.
Algorithms used to obtain Optimal Review Point which minimizes total cost:
For integer point

1. Start from $R_w = 1$ and calculate z
2. Then, Set z_n as integer value.
3. Starting from z_n, calculate minimum total cost and optimum Review point (R_{wn}).
4. Set z_{n-1}, calculate minimum total cost and optimum Review point ($R_{w(n-1)}$).
5. Continue until z_1 and (R_{w1}) until Total cost and optimum Review point obtain.
6. End

4. RESULT AND DISCUSSION

Numerical data's was taken from Madadi et al. (2010) and has been slightly modified. The total cost of the modified data has been calculated with consideration of price discounts.

4.1 Numerical example

Table 1.
Discount price

Discount	Price (P_k)	U_k
0%	90.0	1
1%	89.1	200
2%	88.2	400
3%	87.3	600

Price discount has been set to 1%, 2% and 3% for every minimum order of200, 400 and 600. Based on these amounts and minimum order quantity, we calculated the optimum order quantity to obtain minimum total cost.

4.1.1 Retailer Total Cost

A_j = 100/order; K_j = 1.64; α_j = 100/order;
t_j = 15/truck/order; r_j = 10% ;
VC_j = 100 unit/truck

Table 2.
EOQ and Total cost with discount price

Retailer (j)	μ_j	σL_j	d_{wj}	P_i	Q_i	g_i	Cp_i	CR_i	Cc_i	Ct_i	CT_i
1	857	3	15	87.3	600	6	74816.10	142.83	2661.95	2071.08	79691.97
2	698	3	25	88.2	400	4	61563.60	174.50	1807.39	2792.00	66337.49
3	983	7	20	87.3	600	6	85815.90	163.83	2719.22	3112.83	91811.79
4	687	2	25	89.1	200	2	61211.70	343.50	920.22	2919.75	65395.17
5	786	5	28	88.2	400	4	69325.20	196.50	1836.32	3497.70	74855.72
6	921	7	18	87.3	600	6	80403.30	153.50	2719.22	2640.20	85916.22
Total	**4932**						**433135.80**	**1174.67**	**12664.34**	**17033.57**	**464008.37**
			% Cost/TC				**93.3%**	**0.3%**	**2.7%**	**3.7%**	

Source: Madadi, et. al. 2010. with some modifications

Table 2 shows that total cost for retailers that considers price discounts is lower by 1.37% compared. It means that retailers can save 1.37% of their total cost by considering price discounts.. They can also reduce purchase cost by 2.4% transportation cost by 7.05%, and replenishment cost by 52.37%. However, this policy could in-turn increase their inventory level which results to an increase in their carrying cost up to 118%

Table 3
EOQ and Total cost without price discount

Retailer (j)	Q_i	g_i	Cp_i	CR_i	Cc_i	Ct_i	CT_i	ΔTC	% ΔTC/CT
1	200	2	77130.00	428.50	944.28	2356.75	80859.53	1167.56	1.44
2	200	2	62820.00	349.00	944.28	2966.50	67079.78	742.29	1.11
3	200	2	88470.00	491.50	1003.32	3440.50	93405.32	1593.53	1.71
4	200	2	61830.00	343.50	929.52	2919.75	66022.77	627.60	0.95
5	200	2	70740.00	393.00	973.80	3694.20	75801.00	945.28	1.25
6	200	2	82890.00	460.50	1003.32	2947.20	87301.02	1384.80	1.59
Total			**443880.00**	**2466.00**	**5798.52**	**18324.90**	**470469.42**	**6461.05**	**1.37**
% Cost/TC			**94.3%**	**0.5%**	**1.2%**	**3.9%**			

4.1.2 Warehouse Total Cost

A_w = 80/order; K_w = 1.6; α_w = 100/order;
t_w = 15/truck/order; r_w = 10% ;
VC_w = 125 unit/truck; B_w = 150/stockout;
P_u = 0.95; LT_{sw} = 0.1 year

Table 4
Optimum Review Point and Total Cost

μ_w	σ_w	P_w	R_w	z	Cp_w	CR_w	Cc_w	CT_w	C_{sw}	CT_w
4932	91	60	**0.152**	6	295920	**526.66**	**4165.57**	**12508.23**	**938.12**	**314058.58**

Source: Madadi, et. al. 2010. with some modifications

The above figure shows that the amount of orders from retailers doesn't affect the amount of total cost incurred by a warehouse. On the other hand price discounts offered by warehouse to the retailers could decrease their marginal profit since the price per unit also decreases. But this kind of situation is considerable as the other competitors tend to offer price discount s as well.

4.2 Sensitivity Analysis

4.2.1 Change in carrying cost

To measure the sensitivity of each proposed, we used a variation of carrying cost as fraction of unit cost between ranging from 1% to20%. We tried to obtain the effect of different carrying cost to total cost and number of order by retailers related to discount price. The graph below shows that the total cost increases as the carrying cost increases.

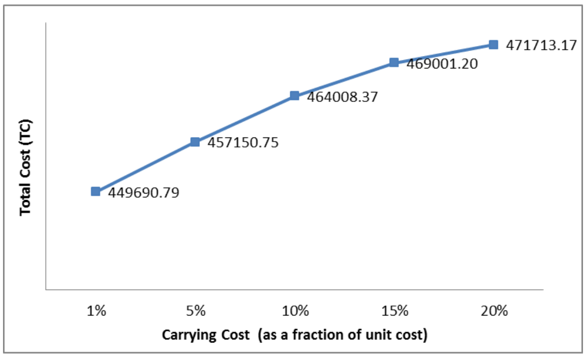

Figure 3
Variation of Carrying Cost

Table 5 shows that carrying cost influence the decision of retailers in selecting the best price discounts. When carrying cost is equal to 1%, retailers selects the higher price discounts. On the other hand, retailers select the lowest discount price offered when carrying cost is equal to 20%.

Table 5
Discount Price in Variation of Carrying Cost

Ret. (j)	1%		5%		10%		15%		20%	
	DP	Q	DP	Q	DP	Q	DP	Q	DP	Q
1	3%	700	3%	600	3%	600	1%	200	1%	200
2	3%	600	2%	400	2%	400	1%	200	1%	200
3	3%	800	3%	600	3%	600	2%	400	1%	200
4	3%	600	2%	400	2%	400	1%	200	1%	200
5	3%	700	2%	400	2%	400	2%	400	1%	200
6	3%	700	3%	600	3%	600	2%	400	1%	200
TC	449691		457151		463919		469001		471713	

4.2.1 Change in replenishment cost

Variation of replenishment cost from 50-200 influence total cost but doesn't influence the policy of retailers in selecting price discounts.

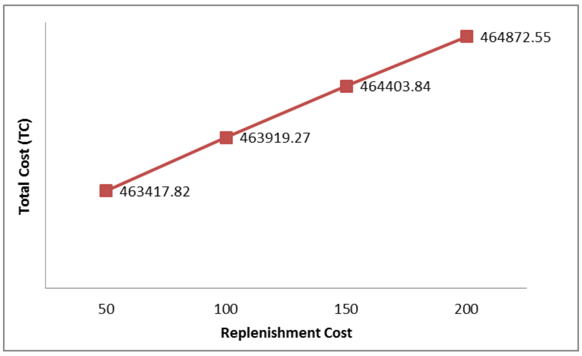

Figure 4
Variation of replenishment Cost

4.2.3 Change in Variable Transportation Cost

Figure 4 illustrates that total cost gradually increases in proportion to the increase in variable transportation cost. However, the range of variation between10 to 25 has no longer impact to the policy of retailers in selecting discount price.

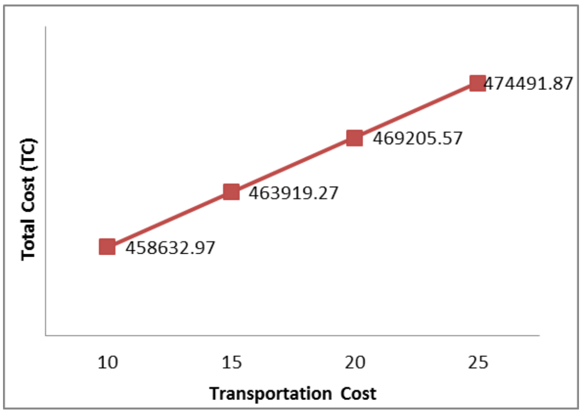

Figure 4
Variation of Variable Transportation Cost

5. CONCLUSION

Equations from Madadi, et. al (2010) has been modified by considering all quantities discount price. The modified equations are different from the original equations because purchase cost has been added. We also modified the solution approach algorithm to optimize the total cost, and derive the optimum order quantity and review point.

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The sensitivity analysis of the parameters related to the total cost were analyzed, and different all quantities discount price ranging from 1-3% has been applied to reduced retailers total cost by 1.37%. We show that a high discount price causes an increase in the carrying cost. All quantities discount price offered to retailers does not influence the total cost in warehouse but in-turn reduces the warehouse gross margin profit. However, in a very competitive market environment, other competitors tend to offer discount prices as well. In order to win the other firms customers, decreasing marginal profit strategy is necessary. Further research may be done to investigate a strategic discount price that a warehouse should offer to its retailers.

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