

CRASHING PROJECT SCHEDULE NETWORK WITH METHODS SELECTION

Ismail H. Asrul

Industrial Engineering Department, Pancasila University, Srengseng Sawah,
Jakarta 12640, Indonesia
E-mail : asrul@univpancasila.ac.id

ABSTRACT

Crashing is a process of expediting project schedule by compressing the total project duration. It is helpful both in executing process, when managers want to anticipate the late project status, and in planning process, when the initial estimation does not meet objective time completion's key stakeholder. Crashing the schedule plan would not only affect the duration of certain task but also the method applied to it indirectly. All methods that can complete the task could have different cost slope. In this paper we want to present a model to select the best method of crashed task to meet minimum cost.

Key words: *Crashing; Cost Slope; Integer Nonlinear Programming.*

1. INTRODUCTION

Projects comprise a series of activities, some of which must be performed sequentially and others that can be performed in parallel with other activities. This collection of series and parallel tasks can be modeled as a network [Stevenson, 2007]. The two best-known networking-planning models are The Critical Path Method (CPM) and The Program Evaluation and Review Technique (PERT). CPM is based on the assumption that project activity times can be estimated accurately and that they do not vary. PERT was developed for the U.S. Navy's Polaris missile project. This was a massive project involving over 3,000 Contractors. Because most of the activities had never done before, PERT was developed to handle uncertain time estimates. As years passed, features that distinguished CPM from PERT have diminished, so in our treatment for crashing here we just use the term CPM.

The fundamental departure of CPM from PERT is that CPM brings the concept of cost more prominently into the planning and control process. When time can be estimated rather well and when costs can be calculated rather accurately in advance (labor and materials for a construction project, for example), CPM may be superior to PERT [Levin, 2007]. Under the CPM system, two time and cost estimates are indicated for

each activity in the network; these two are a normal estimate and a crash estimate. The crash time estimate is the time that would be required if no costs were spared in reducing the project time. Crash cost is the cost associated with doing the job on a crash basis so as to minimize completion time [PMI, 2008].

Moreover, current crashing method copes with single method for each critical activity. This paper wants to present a model to solve the problem that each task can be done by more than one method. Thus it does not only determine the amount of time to reduce to meet minimum crashing cost, but also which method will be used for. Since the process mapping in project management using the Network Diagram, the task that can be done by more than one method (the "or" condition) cannot be seen. The schedule case study will be depicted by using the Functional Flow Block Diagram (FFBD), this condition is illustrated in Figure 2. Figure 1(a)&(b) depict the project schedule case study using Activity On Arc and Activity On Node.

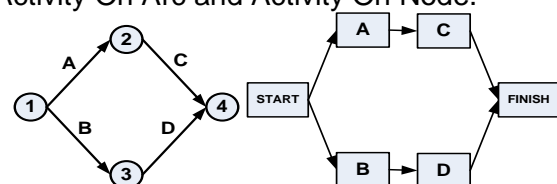


Figure. 1. Network Diagram using (a) Activity on Arc; (b) Activity on Node.

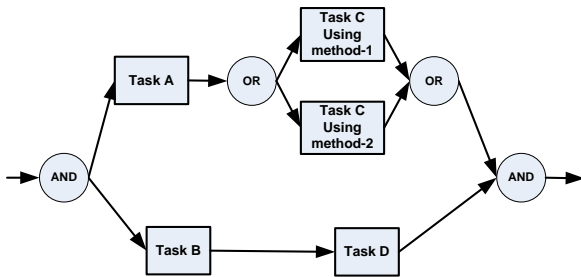


Figure. 2. Network Diagram using FFBD with "or" condition.

To crash a project successfully, we examine the network, note its activities and compare normal costs with crash costs for each activity. Our goal is to find those activities on the Critical Path there time can be cut substantially with minimum extra dollar spent. Our goal is the greatest reduction in project time for the least increase in project cost. But what if the critical path comprises one or more critical activity that have more than one method to complete the activity.

In Figure 1 and 2 can be seen that the Network Diagram cannot accommodate if an activity has more than one method to complete it. There are several models that can describe the "or" condition such as IDEF3, FFBD, but we chose to describe it using FFBD. It can be seen in Figure 2 that the cost of an activity may vary depending on the duration of A and methods used. In dealing with projects possibly involving thousands of such dependency relations, it is no wonder that managers seek effective methods of analysis. The critical questions which would be answered in this paper are:

- "Which activities have to crash and what method has to use to meet minimum project cost with targeted project time completion?"
- "Which activities have to crash and what method has to use to meet minimum project time completion with available fund?"

2. LITERATURE REVIEW

2.1. The General Linear Programming Model of CPM/PERT Network

The solution of this linear programming model will indicate the earliest time of each

node in the network and the project duration [Wallace, 1990].

$$\text{Min } Z = X_n$$

Subject to

$$X_j - X_i \geq t_{ij}$$

$$X_i, X_j \geq 0$$

for all activities $i \rightarrow j$

2.2. Linear Programming Model of Crashing

The objective function is to reduce the project duration at the minimum possible crash cost. The objective function coefficients are the activity cost slope, the variable y_{ij} indicate the number of time each activity will be reduced [Taylor III, 1999].

$$\text{Min } Z = \sum_i^m \sum_j^n CS_{ij} \cdot Y_{ij}$$

Subject to

$$Y_{ij} \leq r_{ij} \quad \text{for all activities } i \rightarrow j$$

$$X_i + t_{ij} - Y_{ij} \geq X_j \text{ for all activities } i \rightarrow j$$

$$X_n \leq T$$

$$X_i, X_j, Y_{ij} \geq 0$$

The relationship between crash cost and crash time is linear, then an activity can be crash by \$ Cost Slope per time. Consider that a cost slope is negative differentiated from the equation of its cost of time/duration function [5]. If the cost of time function as an activity like $CT_{ija} = f(t_{ij})$ where t_{ij} as working time of an activity, then the rate of CT_{ija} is $\frac{dCT_{ija}}{dt_{ij}}$. Then we know that the rate of A activity is $\frac{dCT_{121}}{dt_{12}} = -1500$. It means that the value of -1500 will decrease \$1,500,000 if the working time of the project (t_{ij}) increases 1 week. Look at Figure 3.

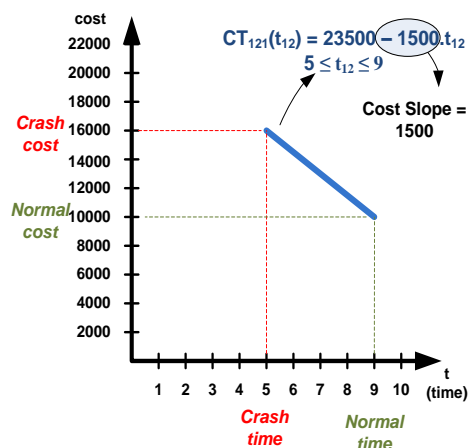


Figure 3a. Cost of Time/Duration Function;

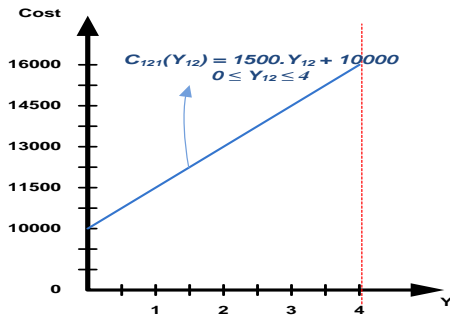


Figure. 3b. Cost of Time to reduce Function.

If the cost of reduction time function is described as $C_{ija} = f(Y_{ij})$ where Y_{ij} as a symbol of time reducing an activity, then $C_{ija} = \text{normal cost} + \left(-\frac{dCT_{121}}{dt_{12}}\right) \cdot Y_{ij}$. Figure 2(b) shows the illustration of accelerating cost of activity A. In other word, we can say that a cost slope is differentiated from the cost of time to reduce function [Ismail, 2009]. Cost slope Activity A using method-1 is differentiated from accelerating cost of activity A $(CS_{121}) = \left(\frac{dC_{121}}{dY_{12}}\right) = 1500$.

2.3. Functional Flow Block Diagram

A Functional Flow Block Diagram (FFBD) is a multi-tier, time-sequenced, step-by-step flow diagram of a system's functional flow. The FFBD notation was developed in the 1950s, and is widely used in classical systems engineering. FFBDs are one of the classic business process modelling methodologies, along with flow charts, data flow diagrams, control flow diagrams, Gantt charts, PERT diagrams, and IDEF [DoD, 2001].

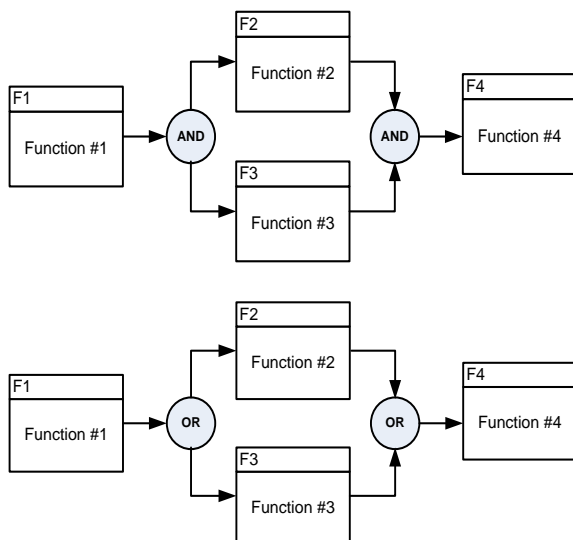


Figure. 4. (a) "AND"; (b)"OR" condition.

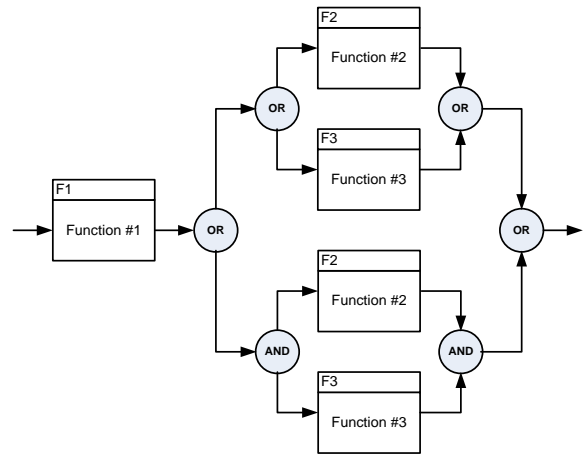


Figure 5. "AND" and "OR" condition in FFBD

The FFBD is functionally oriented—*not* solution oriented. The process of defining lower-level functions and sequencing relationships is often referred to as functional decomposition. It allows traceability vertically through the levels. It is a key step in developing the functional architecture from which designs may be synthesized.

3. RESEARCH METHOD

Problem that state in introduction section can be solved by using existing methodology. The steps are:

Step 1 - Define activities, The process of identifying the specific actions to be performed to produce the project deliverables.

Step 2 - Sequence activities, The process of identifying and documenting relationships among activities. Network diagram should be converted to FFBD in order to describe the alternatives method.

Step 3 - Estimate activity resources and Estimate activity durations, The process of estimating the type and quantities of material, people, equipment, and supplies required to perform each schedule activity and approximating the number of work periods needed to complete individual activities with estimated resources. This process is also determining normal time, normal cost, crash time, crash cost, and cost slope.

Step 4 - Develop Schedule, The process of analyzing activity sequences, durations,

resource requirements, and schedule constraints to create the project schedule.

Step 5 - Determine the critical path, The critical path method calculates the theoretical early start and finish dates, and late start and finish dates, for all activities by performing a forward pass analysis and a backward pass analysis through the project schedule network paths. Activities on a critical path are called "critical activities." A critical path is normally characterized by zero total float/slack across the critical path and network paths can have multiple near critical paths.

Step 6 - Crash the schedule and choose the best method to accomplish the multi-method activities, The output of the model in this paper will determine task to crash, time to reduce, the best method to meet the minimum cost of project.

To develop the proposed model in this paper, several scenarios involving combination of serial parallel task are experimented.

Scenario 1 - project that consists of 2 serial and 2 parallel activities and its cost are illustrated in Figure 6 and 7.

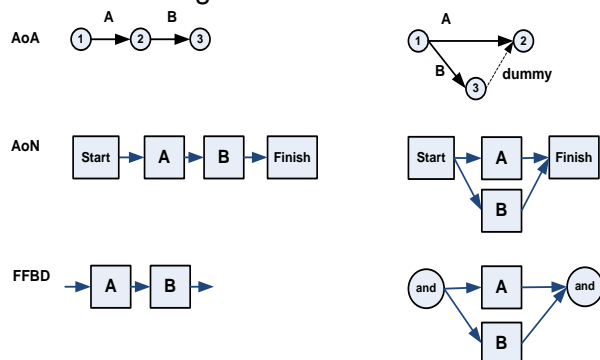


Figure 6. (a) Serial and (b) Parallel Activities.

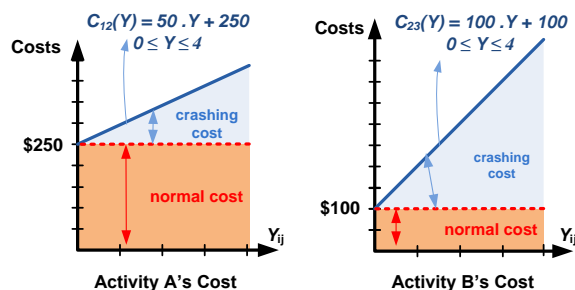


Figure 7. Activities Cost Scenario 1 (a) Cost of Task A; (b) Cost of Task B.

Total Normal Cost of 2 serial activities =
Total Normal Cost of 2 parallel activities =

$$\sum_{i=1}^m \sum_{j=1}^n \text{normal cost} + CS_{ij} \cdot Y_{ij} =$$

$$\sum_{i=1}^m \sum_{j=1}^n \min CT_{ij} + CS_{ij} \cdot Y_{ij} = (\$250 + \$50.0) + (\$100 + \$100.0) = \$350.$$

Scenario 2 - project that consists a task that can be done using 2 alternatives methods (method-1 and method-2) in Figure 8. Method-1's cost and method-2's cost are task A's cost and task B's cost depicted in Figure 7 consecutively.

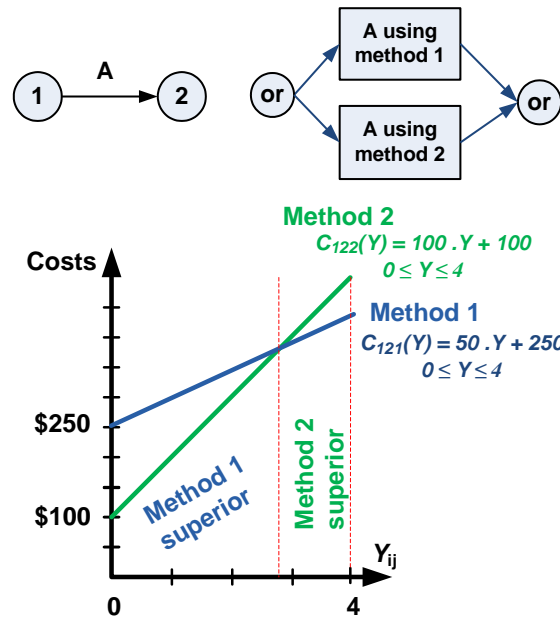


Fig. 8. Scenario 2 illustrated by (a) AoA; (b) FFBD; (c) "or" condition cost functions.

Estimated minimum cost of task A in normal condition = (\$250 + \$50.0) OR (\$100+\$100.0) = \$ 100. Figure 8 shows that the minimum cost in normal condition (Y₁₂=0) is \$100 by using Method-2. To represent the "or" condition above in mathematical model we can put the binary variable, H_{ija}, in it.

$$= (\text{Min } CT_{ij1} + CS_{ij1} \cdot Y_{ij}) \cdot H_{ij1} + (\text{Min } CT_{ij2} + CS_{ij2} \cdot Y_{ij}) \cdot H_{ij2} \quad (1)$$

After Run several experiment scenario we find that Equation above not only can calculate serial, and parallel, but also serial with 'OR' condition. To cope with complex project, we have to redesign by adding the binary variables and constraints that can link the serial-parallel (combination) condition. The objective function is to reduce the project duration, and select the best method

at the minimum possible project completion cost.

The objective function coefficients of the model are the activity cost slopes, the variable y_{ij} indicates the number of time each activity will be reduced and binary variable H_{ija} indicates the binary decision to select the best method to use. The proposed model in this paper is presented (4) below.

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{a_{ij}=1}^{A_{ij}} (\text{Min } CT_{ija} + CS_{ija} \cdot Y_{ij} \cdot H_{ija})$$

Subject to

$$\begin{aligned} \sum_{a_{ij}=1}^{A_{ij}} H_{ija} &= 1, && \text{for all activities } i \rightarrow j \\ Y_{ij} &\leq r_{ij} && \text{for all activities } i \rightarrow j \\ X_i + t_{ij} - Y_{ij} &\geq X_j && \text{for all activities } i \rightarrow j \\ X_n &\leq T \\ X_i, X_j, Y_{ij} &\geq 0 \\ H_{ija} &\text{ binary} \end{aligned} \quad (2)$$

The solution of this integer binary programming model will indicate the earliest time of each node in the network, project cost, time to be reduced, and the method to be used.

Nomenclature

- X_i earliest start event time of node i .
- X_j earliest finish event time of node j .
- t_{ij} time of activity $i \rightarrow j$.
- m number of the last node i in the network.
- n number of the last node j in the network.
- Y_{ij} amount activity $i \rightarrow j$ can be crashed.
- r_{ij} number of the range amount activity $i \rightarrow j$ can be crashed.
- T targeted project completion time.
- CS_{ij} Cost Slope of activity $i \rightarrow j$.
- a_{ij} Number of methods alternatives of activity $i \rightarrow j=1,2,\dots, A_{ij}$
($A_{ij} = 1$, If a Task has only 1 method)
- CS_{ija} Cost Slope of activity $i \rightarrow j$ using method a .
- CT_{ija} Cost of activity $i \rightarrow j$ for t_{ij} days duration using method a .
- C_{ija} Cost of activity $i \rightarrow j$ for Y_{ij} days crashed using method a .

4. CASE STUDY

To demonstrate the proposed approach, a case study involving serial, parallel, and alternatives methods activities combination is introduced in Table 1.

Table 1. Time, Method, and Cost for Case-Study

| Activity | A_{ij} | Method | Time (weeks) | | Cost (\$) | |
|----------|----------|----------|--------------|-------|-----------|--------|
| | | | Normal | Crash | Normal | Crash |
| A | - | Method 1 | | | 10,000 | 16,000 |
| | | Method 2 | 9 | 5 | 8,000 | 20,000 |
| | | Method 3 | | | 9,000 | 15,400 |
| B | - | 1 | 4 | 2 | 4,000 | 6,000 |
| C | A | 1 | 5 | 3 | 7,500 | 10,000 |
| D | B | 1 | 8 | 5 | 14,000 | 21,500 |
| E | C | 1 | 3 | 2 | 9,000 | 18,000 |
| F | D | 1 | 5 | 4 | 1,500 | 4,500 |
| G | E | 1 | 7 | 5 | 3,500 | 6,500 |

4.1. Case Study 1

In normal condition, organization used to complete this kind of project around 24 weeks. But the key stakeholder want the project has to be done in 15 weeks. Which activities have to crash and using what method to meet minimum project cost?

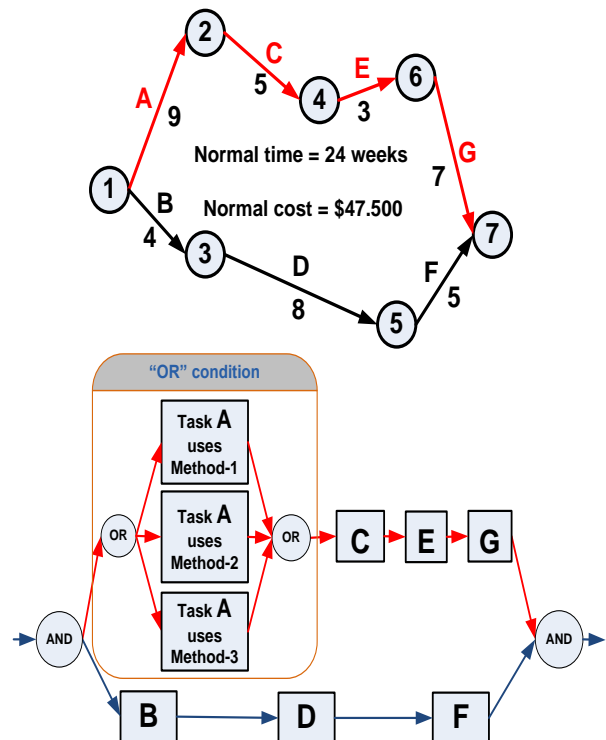


Figure. 9. Case Study's Network Diagram (a) AoA; (b) FFBD.

After Develop Schedule by developing network diagram, we can determine the Cost of reduction time, $F(Y_{ij}) = C_{ij}$, and the slope of every cost (we can see in Table 2). We know from Figure 9 that A-C-E-G is the critical path.

Table 2. Cost of Reduction Time Function and Cost Slope for Each Activity for Case Study

| Activity | Node | Cost of Reduction Time Function C_{ija} | Range | Cost Slope |
|----------|---------|--|------------------------|------------|
| A | i=1;j=2 | $10000 + 1500 \cdot Y_{12}$ | $0 \leq Y_{12} \leq 4$ | 1500 |
| | | $8000 + 3000 \cdot Y_{12}$ | $0 \leq Y_{12} \leq 4$ | 3000 |
| | | $9000 + 1600 \cdot Y_{12}$ | $0 \leq Y_{12} \leq 4$ | 1600 |
| B | i=1;j=3 | $4000 + 1000 \cdot Y_{13}$ | $0 \leq Y_{13} \leq 2$ | 1000 |
| C | i=2;j=4 | $7500 + 1250 \cdot Y_{24}$ | $0 \leq Y_{24} \leq 2$ | 1250 |
| D | i=3;j=5 | $14000 + 2500 \cdot Y_{35}$ | $0 \leq Y_{35} \leq 3$ | 2500 |
| E | i=4;j=6 | $9000 + 9000 \cdot Y_{46}$ | $0 \leq Y_{46} \leq 1$ | 9000 |
| F | i=5;j=7 | $1500 + 3000 \cdot Y_{57}$ | $0 \leq Y_{57} \leq 1$ | 3000 |
| G | i=6;j=7 | $3500 + 1500 \cdot Y_{67}$ | $0 \leq Y_{67} \leq 2$ | 1500 |

The mathematical model as described in Equation (5) can be solved by LINGO 10 software program. This is possible if we write the objective function and all constraints with the role of RHS (all variables must be written in the left side and constant values in the right side of the symbol "=", "<=", ">="). Equation below shows the description of mathematical model for LINGO 10:

$$\begin{aligned} \min &= ((10000+1500 \cdot Y_{12}) \cdot H_{121} + (8000+3000 \cdot Y_{12}) \cdot H_{122} + (9000+1600 \cdot Y_{12}) \cdot H_{123}) \\ &+ (4000+1000 \cdot Y_{13}) + (7500+1250 \cdot Y_{24}) \\ &+ (14000+2500 \cdot Y_{35}) + (9000+9000 \cdot Y_{46}) \\ &+ (1500+3000 \cdot Y_{57}) + (3500+1500 \cdot Y_{67}); \\ &H_{121} + H_{122} + H_{123} = 1; \\ &X_7 = 15; \\ &x_1 = 0; \\ &Y_{12} \leq 4; \\ &Y_{13} \leq 2; \\ &Y_{24} \leq 2; \\ &Y_{35} \leq 3; \\ &Y_{46} \leq 1; \\ &Y_{57} \leq 1; \\ &Y_{67} \leq 2; \\ &x_2 - x_1 + Y_{12} \geq 9; \\ &x_3 - x_1 + Y_{13} \geq 4; \\ &x_4 - x_2 + Y_{24} \geq 5; \\ &x_5 - x_3 + Y_{35} \geq 8; \end{aligned}$$

$$\begin{aligned} &x_6 - x_4 + Y_{46} \geq 3; \\ &x_7 - x_5 + Y_{57} \geq 5; \\ &x_7 - x_6 + Y_{67} \geq 7; \\ &@bin(H_{121}); \\ &@bin(H_{122}); \\ &@bin(H_{123}); \end{aligned}$$

end

Solve-1(LINGO 10):

| Variable | Value | Reduced Cost |
|----------|-----------|--------------|
| Y12 | 4.000000 | 0.000000 |
| H121 | 0.000000 | 600.0000 |
| H122 | 0.000000 | 4600.0000 |
| H123 | 1.000000 | 0.000000 |
| Y13 | 2.000000 | 0.000000 |
| Y24 | 2.000000 | 0.000000 |
| Y35 | 0.000000 | 1500.0000 |
| Y46 | 1.000000 | 0.000000 |
| Y57 | 0.000000 | 2000.0000 |
| Y67 | 2.000000 | 0.000000 |
| X7 | 15.000000 | 0.000000 |
| X1 | 0.000000 | 0.000000 |
| X2 | 5.000000 | 0.000000 |
| X3 | 2.000000 | 0.000000 |
| X4 | 8.000000 | 0.000000 |
| X5 | 10.000000 | 0.000000 |
| X6 | 10.000000 | 0.000000 |

Figure 10. Result Problem-1 using LINGO10

The values of the decision variables and constraints from LINGO's report form problem-1 are:

$Y_{12}=4, H_{121}=0, H_{122}=0, H_{123}=1, Y_{13}=2, Y_{24}=2, Y_{35}=0, Y_{46}=1, Y_{57}=0, Y_{67}=2, X_1=0, X_2=5, X_3=2, X_4=9, X_5=10, X_6=10, X_7=15$. And the value of the objective function (the total of normal + crashing cost) is \$ 71,400.

It means to meet 15 weeks time completion we have to reduce Task A, B, C, E, G for 4, 2, 2, 1, and 2 days consecutively. And Task A has to use Method-3 to reach minimum project cost.

The experiments that had done using 10 scenarios from crashing the project for 1 day to maximum crashing the project (9 days) are showed in Table 3. Moreover the maximum target time completion was 15 days. 14 days of targeted project time⁽⁵⁾ completion will meet an unfeasible condition. We can see all result including all variables and objective in Table 3.

Table 3. Result of the experiments

| Task | Var | X7=23 | X7=22 | X7=21 | X7=20 | X7=19 | X7=18 | X7=17 | X7=16 | X7=15 |
|-------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Z | | 48,750 | 50,000 | 51,500 | 53,000 | 55,600 | 57,200 | 58,800 | 68,800 | 71,400 |
| A | X1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | X2 | 9 | 9 | 9 | 9 | 8 | 7 | 6 | 6 | 5 |
| | Y12 | 0/4 | 0/4 | 0/4 | 0/4 | 1/4 | 2/4 | 3/4 | 3/4 | 4/4 |
| f12=9 | H121 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | H122 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | H123 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| B | X1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| f13=4 | X3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 2 |
| | Y13 | 0/2 | 0/2 | 0/2 | 0/2 | 0/2 | 0/2 | 0/2 | 1/2 | 2/2 |
| C | X2 | 9 | 9 | 9 | 9 | 8 | 7 | 6 | 6 | 5 |
| f24=5 | X4 | 13 | 12 | 12 | 12 | 11 | 10 | 9 | 9 | 8 |
| | Y24 | 1/2 | 2/2 | 2/2 | 2/2 | 2/2 | 2/2 | 2/2 | 2/2 | 2/2 |
| D | X3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 2 |
| f35=8 | X5 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 11 | 10 |
| | Y35 | 0/3 | 0/3 | 0/3 | 0/3 | 0/3 | 0/3 | 0/3 | 0/3 | 0/3 |
| E | X4 | 13 | 12 | 12 | 12 | 11 | 10 | 9 | 9 | 8 |
| f46=3 | X6 | 16 | 15 | 15 | 15 | 14 | 13 | 12 | 11 | 10 |
| | Y46 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 1/1 | 1/1 |
| F | X5 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 11 | 10 |
| f57=5 | X7 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
| | Y57 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 | 0/1 |
| G | X6 | 16 | 15 | 15 | 15 | 14 | 13 | 12 | 11 | 10 |
| f67=7 | X7 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
| | Y67 | 0/2 | 0/2 | 1/2 | 2/2 | 2/2 | 2/2 | 2/2 | 2/2 | 2/2 |

Based on the resume of result presented in Table 3, it is shown that if we want to crash the project for 1 or 2 days we should crash activity C for 1 or 2 days respectively. If we were seeing Figure 8 and Table 2 above it tells us that activity C was the lowest cost slope of critical activities. For example Y₂₄ associated task C with for 23 day project time completion, it is showed 1/2 that means need 1 day crashing from 2 days possible time to reduce. Task D and F are not chosen because they have the biggest cost slope among others and the other critical path, A-C-E-G, has met the maximum time to reduce. We know, from figure 11, that crashing the project for 7 day makes all critical path critical. Figure 11 shows that there is no need to compress an activity for 24 days project time completion.

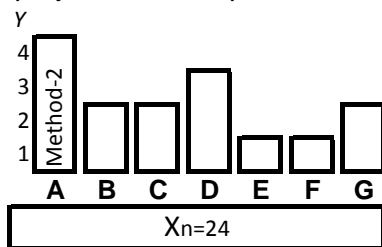


Figure 10. Amount of crashing time for normal time completion.

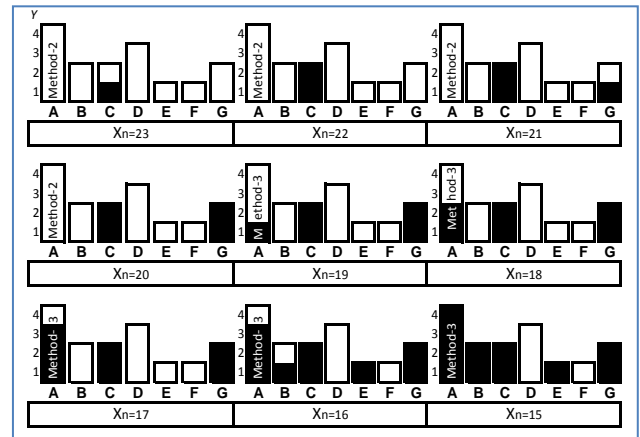


Fig. 11. Amount of crashing time and the best method for each targeted time completions.

Since the range of Y for activity C is 0 to 2 days (Y₄₆) make activity G, the 2nd lowest cost slope in critical path, has to crash 1 day and activity C maximally if we would finish the project in 21 day. For Method that is used to activity A is also shown by Table 3. It is told that if the project has to crash for 1 to 4 days Method 2 will be the best method to use. We can see the resume in Figure 12.

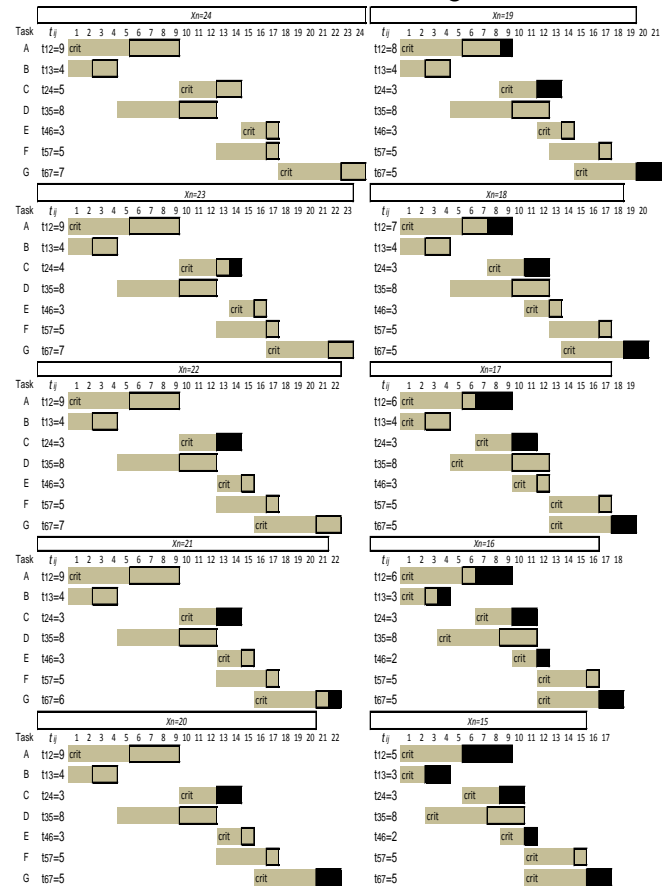


Fig. 12. Gantt chart model for each targeted time completions.

Crashing the project for 5 to maximum crash, 9 days, makes method 3 the best one to complete Task A. And all the paths will be coming critical path if the targeted time completion is 17 days or lesser. It is presented in Figure 12 and 13. Crashing the project for 10 days or greater will demonstrate the unfeasible result. We can see in Figure 13 that A-C-E-G critical path has reached maximum time to reduce. It makes the project can be only crashing maximal to 9 days.

After demonstrating the ability finding the best time to reduce and method to meet the minimum project cost by giving targeted time completion, other question was arising: "How to optimize the time crashing project with available fund?". This question is also can be used to validate the model.

4.2. Case-Study 2

"How many days the optimal project time completion with \$ 53,000 available fund?". The model solved using LINGO 10 is shown in Equation (6).

```

min= X7;
((10000+1500*Y12)*H121
+(8000+3000*Y12)*H122
+(9000+1600*Y12)*H123)
+(4000+1000*Y13) +(7500+1250*Y24)
+(14000+2500*Y35) +(9000+9000*Y46)
+(1500+3000*Y57)
+(3500+1500*Y67)=53000;
H121+H122+H123=1;
x1=0;
Y12<=4;
Y13<=2;
Y24<=2;
Y35<=3;
Y46<=1;
Y57<=1;
Y67<=2;
x2-x1+Y12>=9;
x3-x1+Y13>=4;
x4-x2+Y24>=5;
x5-x3+Y35>=8;
x6-x4+Y46>=3;
x7-x5+Y57>=5;
x7-x6+Y67>=7;
@bin(H121);
@bin(H122);
@bin(H123);
end
    
```

| | | |
|-------------------------------|-----------|--------------|
| Local optimal solution found. | | |
| Objective value: | | 20.000000 |
| Extended solver steps: | | 0 |
| Total solver iterations: | | 22 |
| Variable | Value | Reduced Cost |
| X7 | 20.000000 | 0.000000 |
| Y12 | 0.000000 | 0.000000 |
| H121 | 0.000000 | 0.6666667 |
| H122 | 1.000000 | 0.000000 |
| H123 | 0.000000 | 0.3333333 |
| Y13 | 0.000000 | 0.3333333 |
| Y24 | 2.000000 | 0.000000 |
| Y35 | 0.000000 | 0.8333333 |
| Y46 | 0.000000 | 2.000000 |
| Y57 | 0.000000 | 1.000000 |
| Y67 | 2.000000 | 0.000000 |
| X1 | 0.000000 | 0.000000 |
| X2 | 9.000000 | 0.000000 |
| X3 | 4.000000 | 0.000000 |
| X4 | 12.000000 | 0.000000 |
| X5 | 12.000000 | 0.000000 |
| X6 | 15.000000 | 0.000000 |

Solve-2(LINGO-10):

Figure 14. Result Problem-2 using LINGO 10

The values of the decision variables and constraints from LINGO's report, in Figure 11, form problem-2 are:

$X_7=20$, $Y_{12}=0$, $H_{121}=0$, $H_{122}=1$, $H_{123}=0$, $Y_{13}=0$, $Y_{24}=2$, $Y_{35}=0$, $Y_{46}=0$, $Y_{57}=0$, $Y_{67}=2$, $X_1=0$, $X_2=9$, $X_3=4$, $X_4=12$, $X_5=12$, $X_6=15$.

And the value of the objective function (the optimal project time completion with \$53000 available funds) is 20 days. It means that we could complete the project for 20 days if we have \$53,000 by crashing activities C and G for 2 days each activity. This identical result that can be seen in Table 3 shows the validation of this model to cope with problems in this paper.

5. IMPLEMENTATION

Model implementation covers a broad spectrum of decision making in planning and executing phase. The approach describe in this paper has been successfully implemented in solving selection problems. (6) In project planning, we know that "or" conditions are very familiar. For example the condition for selecting vendor that usually using "make or buy analysis". But the downside is the "make or buy analysis" selecting individually the alternative without concerning the schedule. This paper demonstrates that there is a relationship between cost and time. Thus, making the

decision related to time or cost have to concern both factors.

6. CONCLUSIONS

Method selection in project is cannot characterized by existing process modeling, Network Diagram. Several modeling approaches could be used to describe this "or" condition, like IDEF0, and FFBD. In this paper Functional Flow Block Diagram is used to describe the conditions.

In this study, the novel "or" condition that consist of several alternatives methods combining the schedule planning has been proposed to optimize the project cost. Integer binary programming can be used to optimize the schedule, cost, and method selection simultaneously. The model demonstrates making decision by choosing the lowest cost slope of critical activities to reduce the project optimally. The minimum time completion of a project is the longest critical path that has crashed maximally, in this case study is A-C-E-G = 15 days. Regarding nonlinear form of the model, this research using LINGO 10 to solve the problems.

There are several additional research needs in implementing the model in executing and monitoring and controlling phase where the random/uncertainty condition will be faced. This random condition, like performance, needs deep research in simulation rather than analytical deterministic approaches.

7. REFERENCES

- (a) Levin, I. Richard, et al, (2007), "Quantitative Approaches to Management", Eighth Edition, New York: McGRAW HILL, INC.
- (b) Stevenson, William J., (2007), "Production and Operation Management", 3rd Edition, Tokyo: Toppan Company, Ltd.
- (c) Wallace R.C., (1990), "History of Project Management Applications in the Utility Industry," Project Management Journal 21, no.3 (September): 5-10.
- (d) Taylor III, Bernard W, (1999), "Introduction to Management Science", 6th ed. Prentice-Hall International.
- (e) Ismail, A.H., (2009), "A Study of The Basic Concept of Crashing CPM Network using Derivative Marginal Cost in Linear Programming", 3rd International Seminar on Industrial Engineering and Management.
- (f) PMI, (2008). "A Guide To The Project Management Body of Knowledge", 4th Ed. Project Management Institute, Inc.
- (g) Department of Defense, (2001), "Systems Engineering Fundamentals", The Defense Acquisition University Press Fort Belvoir, Virginia 22060-5565.